Homework 13 in Optimization in Engineering

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Exercise 1. (Newton method with equality constraint) Let

$$\begin{array}{rl} \text{minimize} & f(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \end{array}$$

be an optimization problem with

$$f(\boldsymbol{x}) = \frac{1}{2}x_1^2 \exp(x_3) + \frac{x_1^3}{3} - \frac{1}{2}(x_2+1)^2 + 3x_1x_3 + a(x_2+x_3+1),$$
$$\boldsymbol{x}^T = (x_1, x_2, x_3) \in \mathbb{R}^3, a \in \mathbb{R}_+,\ \boldsymbol{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & -3 \end{pmatrix},\ \text{and } \boldsymbol{b} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}.$$

(a) Reformulate the problem such that it is an unconstrained optimization problem.

(b) When is the reformulated problem convex?

(c) Solve the problem applying a pure Newton method with step size $t = 1$, $a = 2$ and $(\boldsymbol{x}^{(0)})^T = (1, -2, 0)$. What problem is solved for arbitrary parameter a?

(d) Now utilize exact line search in the Newton method for solving the problem. How many iterations do you need for $\varepsilon = 10^{-6}$.

Exercise 2. (Adding a quadratic term in Newton method with equality constraint) Let $\boldsymbol{Q}$ be a matrix, then the problem

$$\begin{array}{rl} \text{minimize} & f(\boldsymbol{x}) + (\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b})^T \boldsymbol{Q} (\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}) \\ \text{subject to} & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \end{array}$$

is equivalent to

$$\begin{array}{rl} \text{minimize} & f(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \end{array}$$

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(a) Show that the Newton steps for both problems are equal.