Problem 1. (Separation Theorem) Complete the proof of the Separation Theorem (Theorem 2.1 in the lecture notes), which is proved for a special case in the lecture. Show that a separating hyperplane exists for two disjoint convex sets $C$ and $D$.

Hints.

- You can use the result proved in Theorem 2.1 in the lecture, i.e., that a separating hyperplane exists when there exist points in the two sets whose distance is equal to the distance between the two sets.

- If $C$ and $D$ are disjoint convex sets, then the set $\{x - y \mid x \in C, y \in D\}$ is convex and does not contain the origin.

Problem 2. (Supporting hyperplanes) Represent each of the following closed, convex sets $C \subseteq \mathbb{R}^2$ as an intersection of halfspaces.

a) $C = \{x \in \mathbb{R}^2 \mid x_2 \geq e^{x_1}\}$.

b) $C = \{x \in \mathbb{R}_{>0}^2 \mid x_1 x_2 \geq 1\}$.

Problem 3. (Converse supporting hyperplane theorem) Suppose the set $C$ is closed, has nonempty interior, and has a supporting hyperplane at every point in its boundary. Show that $C$ is convex.