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Tutorial 7

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Problem 1. (Norm reformulation) Linear optimization problems are convex optimization problems for which the objective function and all constraint functions are affine. Let $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ and $\boldsymbol{b} \in \mathbb{R}^m$ be given. Recall that for $\boldsymbol{x} \in \mathbb{R}^n$,

$$||\boldsymbol{x}||_\infty = \max_{i=1,\dots,n} |x_i| \quad \text{and} \quad ||\boldsymbol{x}||_1 = \sum_{i=1}^{n} |x_i|$$

holds. Find an equivalent linear formulation for the following optimization problems.

a) minimize $||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_\infty$

b) minimize $||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_1$

c) minimize $||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_1$ subject to $||\boldsymbol{x}||_\infty \leq 1$

d) minimize $||\boldsymbol{x}||_1$ subject to $||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_\infty \leq 1$

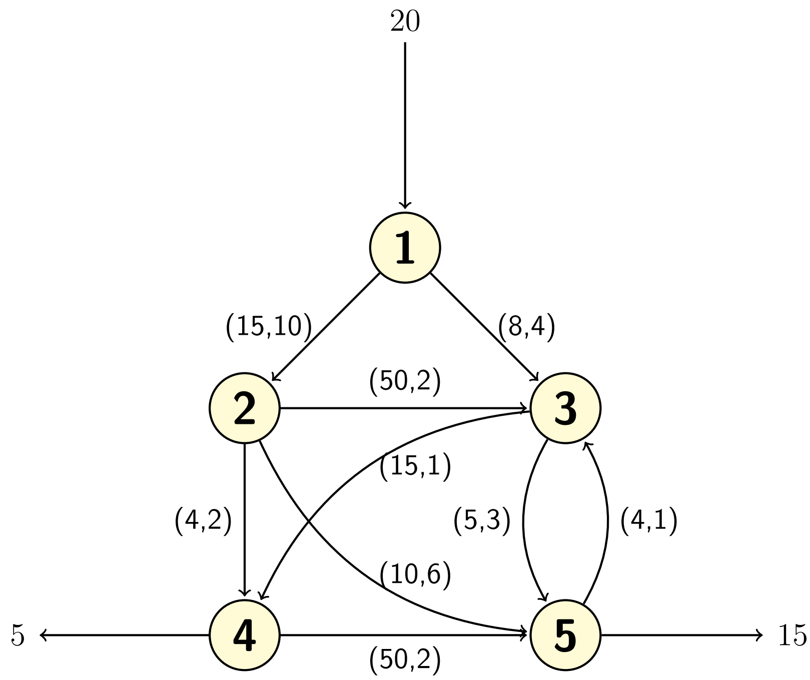
e) minimize $||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_1 + ||\boldsymbol{x}||_\infty$

Problem 2. (Network flow problem) Consider a network of n nodes. The variables in the problem are the flows on each link, where x_{ij} denotes the flow from node i to node j. The cost of the flow along the link from node i to node j is given by $c_{ij}x_{ij}$, where c_{ij} is given. Each link flow x_{ij} is also subject to a given lower bound l_{ij} and an upper bound u_{ij}.

The external supply at node i is given by b_i, where $b_i > 0$ means that an external flow enters the network at node i, and $b_i < 0$ means that at node i, an amount of $|b_i|$ flows out of the network. We assume that $\boldsymbol{1}^T\boldsymbol{b} = 0$, i.e., the total external supply equals total external flow. At each node we have conservation of flow through the network: the total flow into node i along links and the external supply, minus the total flow out along links, equals zero.

The problem is to minimize the total cost of flow through the network, subject to the constraints described above.

a) Formulate this problem as a linear optimization problem.



b) Find the minimum total cost of flow for the network in the graph above (by means of the cvx-package in Matlab). The capacity u_{ij} and the cost of the flow c_{ij} are given next to each link as (u_{ij}, c_{ij}). All lower bounds l_{ij} are assumed to be 0. External supply and flows are shown at nodes 1, 4 and 5 with corresponding direction and amount of the flows.

c) Is the solution unique?