Problem 1. (One-dimensional trust region problem) Consider the one-dimensional, real-valued trust region problem.

\[ \begin{align*} 
\text{minimize} & \quad ax^2 + 2bx \\
\text{subject to} & \quad x^2 \leq 1. 
\end{align*} \]

a) Determine all pairs \((a, b)\) for which the problem is non-convex.

In the following the problem shall be non-convex.

b) Calculate the dual function \(L_D(\lambda)\)

c) Give the optimal parameter \(\lambda^*\) which maximizes \(L_D\) and the corresponding value \(d^*\).

d) Show that the optimal value of the primal problem \(p^*\) equals \(d^*\).

Problem 2. (Dual problem bounds) For the following optimization problems with optimization variable \(x \in \mathbb{R}^2\), compute the dual problem and the maximum lower bound \(d^*\) for the optimal value \(p^*\).

a) \[ \begin{align*} 
\text{minimize} & \quad 2x_1^2 + 8x_2^2 \\
\text{subject to} & \quad 3x_1 + 6x_2 = 10 
\end{align*} \]

b) \[ \begin{align*} 
\text{maximize} & \quad 2x_1x_2 \\
\text{subject to} & \quad x_1^2 + x_2^2 = 1 
\end{align*} \]

Remark: Convert problem (b) into a minimization problem first.