Problem 1. (Optimality conditions) Consider the optimization problem

\[
\begin{align*}
    &\text{minimize } x_1^2 + x_2^2 \\
    &\text{subject to } (x_1 - 1)^2 + (x_2 - 1)^2 \leq 1, \\
    &\quad (x_1 - 1)^2 + (x_2 + 1)^2 \leq 1
\end{align*}
\]

with variable \( \mathbf{x} \in \mathbb{R}^2 \).

a) Sketch the feasible set and level sets of the objective. Find the optimal point \( \mathbf{x}^* \) and the optimal value \( p^* \).

b) Give the expression of the associated Lagrangian and state the KKT conditions. Do there exist Lagrange multipliers \( \lambda_1^* \) and \( \lambda_2^* \) that prove that \( \mathbf{x}^* \) is optimal?

c) Derive and solve the Lagrange dual problem. Does strong duality hold?