Prioritized Throughput Maximization via Rate and Power Control for 3G CDMA: The Two Terminal Scenario

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Outline

- Motivation : Variable Spread Gain CDMA
- Simple Model for Single-Cell CDMA Data
- A Generalized Frame-Success Function
- Throughput equation is defined and optimization is performed
  - Interior "stationary" point (all partial derivatives set to zero) is sought. Second order conditions (SOC) are checked.
  - Boundary stationary point is sought in which bit rate of "important" user is pre-set as high as feasible. SOC are checked.
  - Boundary stationary point is sought in which bit rate of both users are pre-set at highest feasible level. SOC are checked.
- Related/future work

Spread Gain : $G_i = R_C/R_i$ (Chip_rate / bit_rate) ; $G_0 = R_C/R_{MAX}$

γ_0 solves $xf'(x)=f(x)$; γ_{00} solves $x^2 f'(x)=K(G_0)^2/\beta$; β : priority

Motivation : VSG-CDMA

- Modern (3G) wireless nets are expected to accommodate terminals operating at very different data transmission rates.
- Variable Spreading Gain CDMA can accommodate terminals operating at dissimilar bit rates
- In a VSG CDMA system, chip rate is common, but each terminal's spreading (processing) gain is the ratio of the common chip rate to the terminal's bit rate

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CDMA Single Cell Data Comm.

- N transceivers send data to a base station
- R_c : chip rate ; R_i : data rate ; $G_i=R_c/R_i$: Proc. Gain
- $f_s(\gamma_i)$: probability of correct reception of a packet
- $\gamma_i = G_i\alpha_i$ is the SIR with α_i the CIR given by

$$\alpha_i = \frac{h_i P_i}{\sum_{i \neq j} h_j P_j + \sigma^2} = \frac{Q_i}{\sum_{i \neq j} Q_j + \sigma^2}$$

- h_i : "gain" factor
- $h_i P_i := Q_i$: received power

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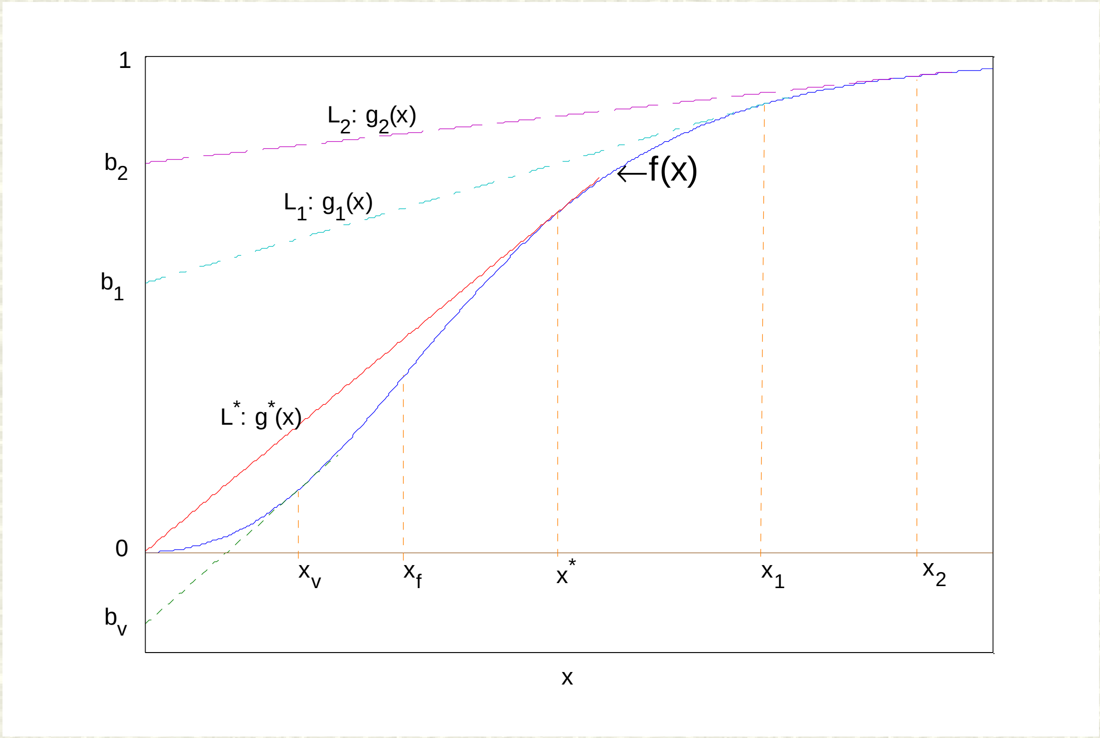
General Frame-Success Function

General S-shaped Frame-Success Function



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Objective Function

- Want to maximize network weighted throughput: $\sum \beta_i R_i f_s(G_i \alpha_i)$
- β_i is a priority weight
- Find for each active user, an optimal power level AND an optimal bit rate
- Power levels determined through optimal power ratios, α_i (CIR); and bit rates determined through optimal processing gains (G_i)
- CIR need to be constrained so that they lead to feasible power levels. For 2-user interference-limited system, $\alpha_1 = Q_1/Q_2 = 1/\alpha_2$ thus $\alpha_1\alpha_2 = 1$
- Each G_i must exceed certain $G_0 \geq 1$ ($R_i \leq R_M \leq R_c$)

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Optimization Model

$$\text{Maximize}\quad \frac{f\left(G_1\alpha_1\right)}{G_1}+\beta\frac{f\left(G_2\alpha_2\right)}{G_2}$$

$$\text{subject to}\quad \alpha_1\alpha_2=1$$

$$G_1 \geq G_0$$

$$G_2 \geq G_0$$

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First-Order Necessary Optimizing Cond.

Augmented Objective Function:

$$\phi(G_1, G_2, \alpha_1, \alpha_2) =$$
$$\frac{f(G_1\alpha_1)}{G_1} + \beta\frac{f(G_2\alpha_2)}{G_2} + \lambda(1-\alpha_1\alpha_2) + \mu_1(G_0 - G_1) + \mu_2(G_0 - G_2)$$

First-Order Necessary Optimizing Conditions (FONOC):

$$\begin{bmatrix} \dfrac{\gamma_1 f'(\gamma_1) - f(\gamma_1)}{G_1^2} - \mu_1 \\ \dfrac{\beta(\gamma_2 f'(\gamma_2) - f(\gamma_2))}{G_2^2} - \mu_2 \\ f'(\gamma_1) + \lambda\alpha_2 \\ \beta f'(\gamma_2) + \lambda\alpha_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{with} \begin{cases} \alpha_1\,\alpha_2 = 1 \\ \mu_1(G_0 - G_1) = 0 \\ \mu_2(G_0 - G_2) = 0 \end{cases}$$

where $\gamma_i = G_i\alpha_i$

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Interior stationary point

- Seek a solution to FONOC in the interior of the feasible region; i.e., suppose $G_1 > G_0$, $G_2 > G_0$ (Lagrangian coefficients $\mu_1 = \mu_2 = 0$)
- This yields closed-form solution:
  $\alpha_1 = 1/\alpha_2 = \sqrt{\beta}$; $G_1\alpha_1 = G_2\alpha_2 = \gamma_0$
- γ_0 solves $xf'(x) = f(x)$. It's unique. (see fig.)
- Consistency requires that $G_1 = \gamma_0/\sqrt{\beta} > G_0$
- Second order conditions indicate this solution is always a "saddle point"
- This allocation is 'fair' : both users enjoy same weighted throughput

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Asymmetric-rate boundary allocation

- Seek a solution to FONOC on the boundary of the feasible region by supposing that $G_2=G_0$ ("favorite" terminal operates at highest feasible bit rate) but that $G_1>G_0$ (Lagrangian coeff. $\mu_1=0$).
- Solution exists whenever there is an x satisfying $x^2 f'(x)/f'(\gamma_0)=(G_0)^2/\beta$.
- The left-hand side of this equation is a "bell-shaped" function. Thus, if $(G_0)^2/\beta$ is "too large" no such x exists. Otherwise, this equation has two solutions.
- Let γ_{00} be the largest of the two values satisfying $x^2 f'(x)= f'(\gamma_0)(G_0)^2/\beta$. All the optimizing values can be determined in terms of γ_{00} and γ_0 .
- γ_{00} gives the optimal SIR of "favorite" user; i.e., $G_2\alpha_2= G_0\alpha_2= \gamma_{00}$. From this, $\alpha_2= \gamma_{00}/G_0 = 1/\alpha_1$.
- The optimal SIR of less important user is γ_0 (preceding slide). This leads to $G_1 =\gamma_0 \gamma_{00} /G_0$. If this value does not exceed G_0 as was presumed, this allocation must be discarded. Thus $G_0<\sqrt{(\gamma_0 \gamma_{00})}$.
- Second order conditions confirm that whenever this solution exists, it is a maximizer

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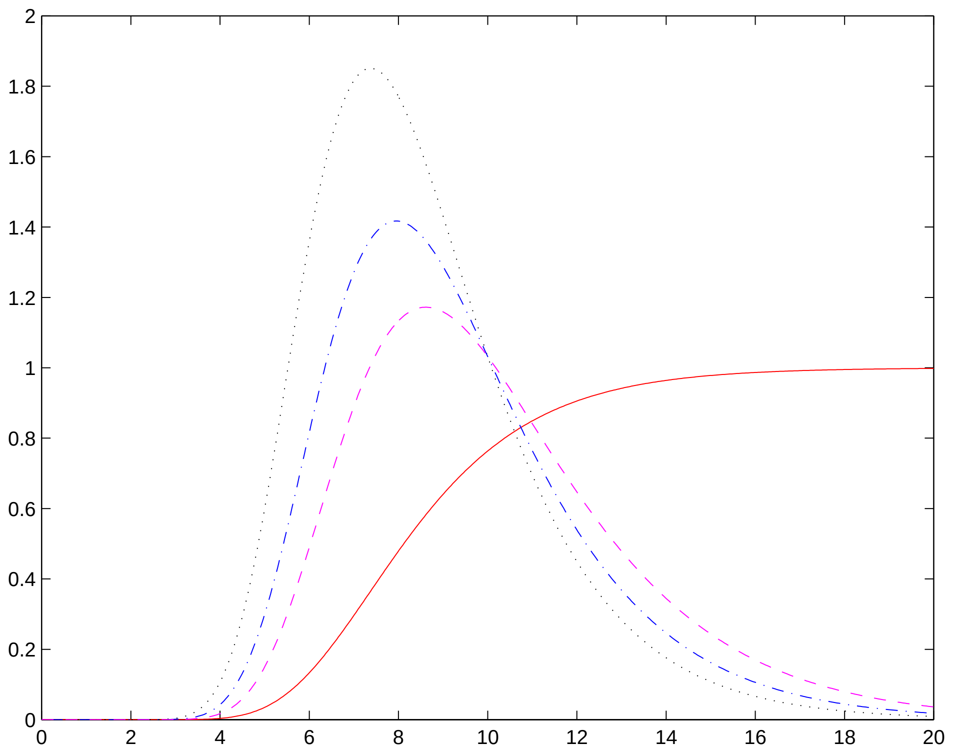
Form of $x^2 f'(x)$ and related functions



Scaled plots of a particular f(x) [solid], f′(x) [dotted], xf′(x) [dashdot], and $x^2 f'(x)$ [dashed]

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“Greedy” Allocation

- Seek a solution to FONOC on the boundary of the feasible region by supposing that $G_2=G_1=G_0$ (both terminals operate at highest feasible bit rate)
- Solution always exists
- Second order conditions indicate that this solution may be a maximizer or a minimizer depending upon system parameters.
- When both terminals are equally important, equal-received power allocation ($\alpha_1=\alpha_2=1$) satisfies FONOC. But this is a maximizer only when G_0 is “large enough” ;i.e., it exceeds a threshold determined by frame-success function (the value at which xf’(x) reaches maximum). Otherwise, allocation is a minimizer.
- Generally, if the solution corresponding to the preceding case ($G_2=G_0$; $G_1>G_0$) does <u>not</u> exist, this solution ($G_2=G_1=G_0$) <u>is</u> a maximizer.

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Summary

- On the maximization of the network weighted throughput in a 2-terminal interference-limited single-cell CDMA :
  - It is always optimal for the important user to transmit at the highest feasible data rate. It may or may not be optimal for the other user to operate at this rate.
  - When $(G_0)^2/\beta$ is "small", only the important user must operate at "full speed". This user's optimal SIR is determined by solving an equation of the form $x^2 f'(x)=K (G_0)^2/\beta$. This optimal SIR immediately determines the optimal power-ratios.
  - The other terminal's data rate is determined so that its SIR (product of its processing gain by its power ratio) equals a channel-determined constant.
  - If maximum permitted bit rate is low enough (G_0 is large enough), it becomes optimal to allow both users to transmit at this fastest rate. Optimal power ratios are then determined by solving certain channel-determined equation.

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Discussion

- On the maximization of the network weighted throughput in a 2-terminal interference-limited single-cell CDMA :
  - Analysis identifies 3 allocations possibly satisfying some optimality criterion: a BALANCED ('fair') allocation, an 'UNFAIR' allocation, and a "GREEDY" allocation.
  - The balanced allocation is always sub-optimal: 'fairness' is expensive!
  - It is always optimal for the favorite terminal to operate at maximum bit rate.
  - When $G_0/\sqrt{\beta}$ is larger than a threshold determined by the physical layer through f, both terminals should be admitted at the maximum permissible data rate.
  - The (data) "speed limit" under which the greedy allocation is optimal Decreases, as the favorite terminal grows in importance.
  - If G_0 is small enough, the greedy allocation actually MINIMIZES the weighted throughput.

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Continuing/future work

- Imposing QoS constraints (minimum throughput per terminal)
- Exploring the 'fairness' issue (saddle point)
- Considering fixed but dissimilar data rates (spread gains)
- Considering noise
- Mobility (location) issues
- Multiple cells
- Extension to "n" terminals

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Related work

- A paper describing the technical details of maximizing S(x)/x with S a general S-curve is available.
- Another work discusses a “robust” generalized QoS measure for wireless data, and a “game” (decentralized algorithm) in which each terminal chooses power to maximize its own QoS service
- See wireless.poly.edu

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