TECHNICAL-ECONOMIC EVALUATION OF SPECTRUM REGIMES: STATIC, DYNAMIC AND UTOPIAN ALLOCATIONS FOR B3G CDMA WITH COEXISTING UWB DEVICES

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ABSTRACT

The technical-economic performances of static (traditional) and auction-driven dynamic radio-spectrum allocation (FSA and DSA), and of an utopian "federated" network (with all spectrum and customers) are compared. Spectrum licensees operate cellular CDMA networks populated by heterogeneous data-downloading terminals, which may coexist with ultra-wide-band (UWB) devices. A terminal has its own data rate, channel state, and "willingness to pay". The key performance figure is the economic value of total bits transferred. The utopian regime always performs best, but is often nearly matched by DSA. The DSA "gain" over FSA ranges from a few percentage points to as high as (in idealised cases) 200%. The technical-economic impact of "high power" UWB under all regimes is considered.

I INTRODUCTION

The radio spectrum is a naturally limited resource of extraordinary social and economic value, as indicated by the enormous sums of money recently raised in UMTS auctions[1]. Spectrum should be managed very efficiently. Yet, traditional (fixed) spectrum allocation (FSA) can be very inefficient when network "loads" vary widely with time and/or space. Dynamic spectrum allocation (DSA) increases efficiency by adjusting to changes in demand. Herein, a dynamic spectrum manager periodically auctions short-term licenses. Each network makes bids, based on current needs. Licenses are issued, which simultaneously expire at the end of a specified short period [2, 3]. We juxtapose this DSA to FSA, and to an utopian federated network (that has all customers and spectrum). The economic value of total bits transferred in a fixed time interval (same as the combined revenues of all networks) measures performance.

Ultra-wide-band (UWB) technology counts among its many virtues the ability to coexist over spectrum assigned to other technologies. Existing regulations make negligible UWB's impact on incumbent networks, but compliant UWB devices are severely range-limited, and hence useful for a very limited class of applications. More powerful UWB devices could be allowed in exchange for some form of "economic mitigation" to incumbents [4]. We consider the technical-economic impact of "higher power" UWB under the 3 regimes.

Below, section II provides essential concepts and results, but a reader new to this approach may need to consult [2, 3]. Section III provides numerical examples, qualitative analysis, and the experimental results (which are the chief contribution over our previous publications). Section IV summarises key results.

II FRAMEWORK AND ANALYSIS

A Physical and behavioural model

(i) N terminals *receive* data *from* a CDMA base station (BS). (ii) $\bar{P}$ is the BS power constraint. The index i identifies a terminal. (iii) R_i denotes data rate (iv) R_C is the chip rate, assumed equal to W, the available bandwidth. (v) Information is sent in M_i-bit packets carrying $L_i < M_i$ information bits (vi) Idealised ARQ is used. (vii) Intra-cell interference is neglected. Thus, the received signal-to-interference ratio (SIR) is $x_i = G_i h_i P_i/\sigma_i^2$ with $G_i := W/R_i$, P_i downlink power, h_i channel gain, and σ_i^2 average noise power. (viii) $F_i(x_i)$ is the packet-success-rate (PSR), but $f_i(x) := F_i(x) - F_i(0)$ replaces $F_i(x)$ for technical reasons. The PSR is assumed "S shaped" (fig. 1).

The average number of information bits successfully transferred over τ is:

$$B_i(x_i) = \tau(L_i/M_i)R_i f_i(x_i) \tag{1}$$

Following [5, Ch. 10], if a terminal (with a "large" monetary budget) pays $c_i(x_i)$ for SIR x_i, it aims to maximise benefit minus cost:

$$\beta_i B_i(x_i) - c_i(x_i) \equiv S_i(x_i) - c_i(x_i) \tag{2}$$

β_i is the "willingness to pay" (wtp), i.e., the "value" of a transferred information bit.

B Technical-Economic CDMA management

Pricing can serve as a tool for both generating revenue, and encouraging efficiency. The network needs (i) a pricing rule, and (ii) a criterion to prioritise terminals when not all can be served. Two key assumptions are (i) the $\beta's$ are known to the network, and (ii) the network can charge an individual price to each terminal. Figure 1 provides the essence of the pricing analysis of [3], whose key conclusions are:

(i) The network chooses for i a price c_i^* obtained as the slope of the only tangent to S_i that goes through the origin. The service SIR is x_i^* (tangency point). If $f_i = f$ for all i, $x_i^* = x^*$ (even if $\beta_i \neq \beta_j$).

(ii) The revenue from i, if served, is:

$$\tau(L_i/M_i)f_i(x_i^*)\beta_i R_i := \tau_i \beta_i R_i \tag{3}$$

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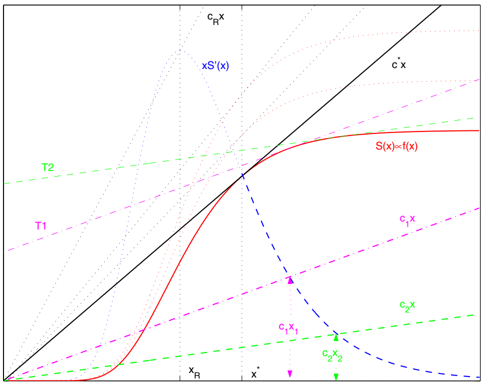


Figure 1: With an SIR of x, $S(x)$ is the terminal's "benefit". $S(x) \propto f(x)$, with f the PSR. x^* is the service SIR, and $c^*x^* = S(x^*)$ what the terminals pays.

For a common link layer, $\tau_i = \tau(L/M)f(x^*) := \tau_0$, a constant that can be set to 1, with convenient units.

(iii) The link configuration with the largest $(L/M)f(x^*)/x^*$ *should* be common.

$x_i = x^*$ for all i may conflict with $\sum P_i = \bar{P}$:

$$\frac{W}{R_i}\frac{h_i P_i}{\sigma_i^2} = x^* \Rightarrow \sum_{i=1}^{N} \frac{R_i}{h_i/\sigma_i^2} \leq \frac{W}{x^*/\bar{P}} \tag{4}$$

Thus, terminal i, if served, consumes :

$$\frac{x^*}{\bar{P}}\left(\frac{R_i}{h_i/\sigma_i^2}\right) := W_0 \frac{R_i}{\hat{h}_i} \tag{5}$$

($\hat{h}_i := h_i/\sigma_i^2$ and $W_0 := x^*/\bar{P}$). A terminal's service priority is determined by its "revenue per Hertz" contribution, which, from (3) and (5), is:

$$\propto \frac{\beta_i R_i}{R_i/\hat{h}_i} = \beta_i \hat{h}_i \tag{6}$$

With $\beta_1 \hat{h}_1 \geq \cdots \geq \beta_N \hat{h}_N$, and I^* the largest index:

$$\sum_{i=1}^{I^*} \frac{R_i}{\hat{h}_i} \leq \frac{W}{W_0} \tag{7}$$

the network should:

(i) serve terminals $1,\ldots,I^*$ each at its *full* data rate; and (ii) admit terminal I^*+1 at the *pertinent fraction* (≥ 0) of its data rate, to use up all the bandwidth.

High-power UWB may raise the noise level of some terminals. "Capacity" (the right of (4)) remains unchanged, but each term on the left may increase ($R_i/\hat{h}_i \equiv R_i\sigma_i^2/h_i$). Thus, I^* could significantly decrease, resulting in fewer terminals served, yet each paying the pre-UWB amount ((3) does *not* depend on σ_i).

C Auction-driven DSA

The multi-unit version of the auction of [6] is used. A brief description follows.

1) Multi-unit Vickrey auction

The available spectrum is divided into K (say 3) bands of width w. Each network submits a K-component vector. A vector (b_1; b_2 ; b_3) means: I offer b_1 for a *total* of 1 band, I offer b_1+b_2 for a *total* of 2 bands, and I offer $b_1+b_2+b_3$ for all 3 bands. If only bidders B1 and B2 submit bids b^1= (5 ; 3 ; 2) and b^2=(4,5 ; 4 ; 1), the assignment goes as follows: 1 band for B1 (5 is top overall bid); the next band to B2 (2nd highest bid is 4,5); last band also to B2 (4, the 2nd component of b^2, is the 3rd highest bid).

A winner's payments depend on the opponent's *losing* bids. Since B1 won only 1 band, he pays the highest *losing* bid submitted by B2, that is, 1. B2 won 2 bands, and pays the sum of the 2 highest *losing* bids submitted by B1, 3+2=5. The auction raises 1+5=6.

2) Network's pricing and bidding

Under this auction, a bidder's "best response" is to bid his "true valuation" of the auctioned object. Thus, the first component of a bid vector should equal the maximal revenue that the network could obtain if it wins a single band (and nothing else). The second component should equal the *extra* revenue it would get if it wins a total of 2 bands, etc. But the network's revenue depends on its own pricing policy, which it must determine along with the bid. Both follow from section B, with W the pertinent amount of spectrum.

For instance, let I_1^* be the last terminal that can be fully served with $W = w$ in (7). Then, the vector's 1st component is $b_1 = \sum_{i=1}^{I_1^*} \beta_i R_i + \delta_1$ with δ_1 the contribution of terminal I_1^*+1. Assuming that the chip rate can be adjusted to match the bandwidth, the network multiplies the right side of (7) by 2, and obtains I_2^* for the new constraint. Thus, the 2nd component is $\sum_{i=1}^{I_2^*} \beta_i R_i + \delta_2 - b_1$ (subtract b_1 to get the *extra* revenue); and so on.

III Performance analysis and experiments

A General experimental framework

We envision 3 "islands" each with a different spectrum regime: (i) FSA (fixed/traditional) (ii) DSA (dynamic, auction-based), and (iii) "utopian" (a single "federated" network manages all spectrum and serves all terminals). Each network in the DSA island has a "mirror image" in the FSA island; and the terminal arrival and service process of the DSA island is "mirrored" in the other 2 islands (if a terminal with parameters β_k, R_k, and h_k arrives at the DSA island, simultaneously, an identical terminal arrives to the appropriate network, elsewhere).

A "market share" vector indicates a network's share of the system's arrival rate. The product of the appropriate arrival rate by the consumption of a "representative" terminal (the ratio of the "average" data rate to the "average" channel gain) yields a statistically "sufficient" amount of spectrum. *Unless otherwise* stated: (i) total system bandwidth, and the system Poisson arrival rate are held constant, as the number of networks change; (ii) market shares are equal, and (iii) FSA spectrum shares are equal. For a specific experiment, spectrum shares are made proportional to market shares.

Convenient units are assumed, so that constants W_0 and τ_0 equal 1. The key performance figure is the "value" of the bits transferred over the inter-auction period (see example in subsection C). The DSA "gain" is the ratio of the value of DSA-transferred bits to the value of FSA-transferred bits (by the same set of arriving terminals). When the transferred-bit values are given, they are "normalised" by dividing them by the average "total value": the triple product of the system arrival rate by the "average" data rate by the "average" wtp.

Immediately before a DSA auction, terminals "arrive" by a Poisson process, and their "service times" precisely equal the inter-auction time. Each arriving terminal needs 3 key parameters: (i) a channel gain obtained from standard path loss calculations, assuming that the distance to the BS is uniformly distributed between 10 and 1000 meters; (ii) a data rate drawn from {5, 3, 1} with respective probability of 1/5, 7/20 and 9/20; (iii) a willingness-to-pay (wtp) value usually drawn from {3, 2, 1} with respective probability of 1/5, 3/10, and 1/2 (unless changed for a specific experiment).

B Qualitative performance analysis

The 2-network best case scenario for DSA is when "loads" alternate "counter-cyclically": when a load is at 1, the other is at 0. Suppose the total spectrum is exactly sufficient for the high load. Each network should get 1/2 the bandwidth, which leaves 1/2 of its demand unfulfilled. Yet, at any moment, a network is idle; thus 1/2 the spectrum is wasted. Under DSA, the busy network would always win all spectrum, and thus satisfy all its demand (the idle network would bid zero). In this idealised case, DSA beats FSA 2-to-1 (and so would the utopian network). In the analogous situation with ν networks, FSA is outperformed ν-to-1.

The above situation is unlikely to materialise. At the opposite end, the networks may face simultaneously the same demand, in *statistical* terms. FSA would divide the spectrum equally among the networks, which seems "optimal". But FSA may still be inefficient: owing to the randomness of the arrival/service process (e.g., the variance of a Poisson process could be very large), over a given small period, there may be significant differences in the networks' "loads". Thus, DSA may still outperform FSA, by allowing a network to win more bands (and "Utopia" may outperform them both), as the experiments show below.

C Simple numerical illustration

Table 1 has the critical data, already sorted by rev/Hertz (convenient units are assumed).

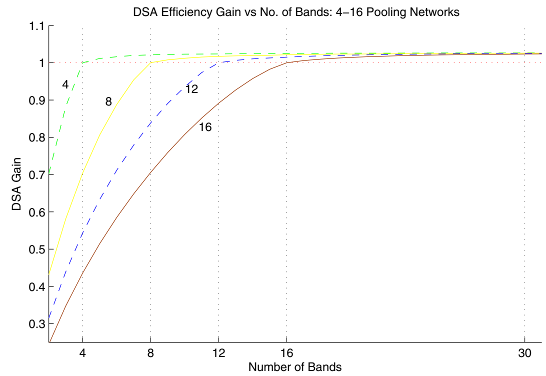


Figure 2: With identical "pooling" networks, DSA efficiency can be dismal if the number of auctioned bands is less than the number of networks. When these 2 numbers are equal, DSA performs as well as FSA. With more bands, DSA modestly outperforms FSA. Adding bands beyond a certain number helps negligibly.

Table 1:

R_i	h_i	β_i	$\beta_i h_i$	R_i/h_i	$\beta_i R_i$
1	1/3	3	1	3	3
1	1/8	2	1/4	8	2
3	1/9	2	2/9	27	6
3	1/10	2	1/5	30	6
3	1/24	4	1/6	72	12

There are 4 bands, each of 29 capacity. With a single band, T1 and T2 can be "fully" served, and T3 admitted at 2/3 its data rate, yielding total revenue of 9. The first 3 terminals plus 2/3 of T4 fit in 2 bands. The 2nd band yields additional revenue of 15-9=6 (2nd component of bid vector). With 3 bands, terminals 1 to 4, plus 19/72 of T5 fit, yielding an *extra* revenue of 20,2-15=5,2. And so on. The vector of bids is $[\ 9 \quad 6 \quad 5{,}2 \quad 4{,}8\ ]$.

D Specific experiments and results

Several experiments are conducted, including:

(i) DSA gain versus number of auctioned bands for various numbers of networks, under 2 scenarios: (a) "pooling" (each network "brings" its own customers and a sufficient amount of spectrum), and (b) "market fragmentation" (system-wide arrival rate and spectrum are held constant regardless of the number of networks). (ii) Gains of DSA and market-share-proportional FSA versus a "symmetry index" a, $0 < a \leq 1$. (iii) Transferred-bit values versus a UWB-motivated "noise amplification" factor. (iv) Transferred-bit values versus "social equality". For reader's convenience, additional details are given inside the figure captions.

IV Discussion

We have compared 3 spectrum regimes: traditional, (auction-based) DSA, and "utopian" (a single network with all spectrum and customers). All networks are UMTS-like, operating

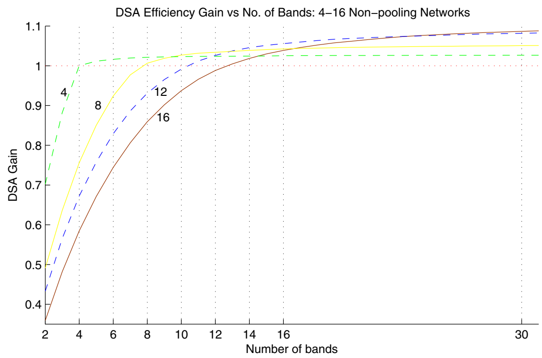


Figure 3: Under "market fragmentation" (system spectrum and arrival rate held constant at the level of the 4-network system of fig. 2) DSA gain increases with the number of bands and with the number of identical networks.

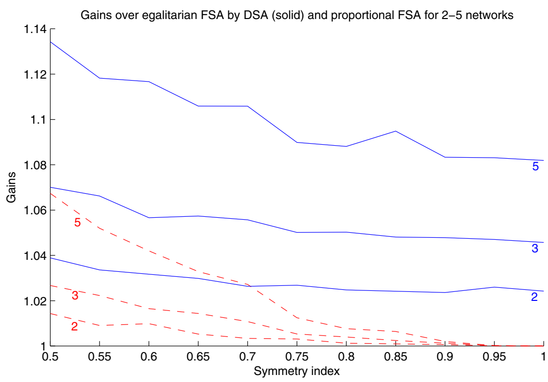
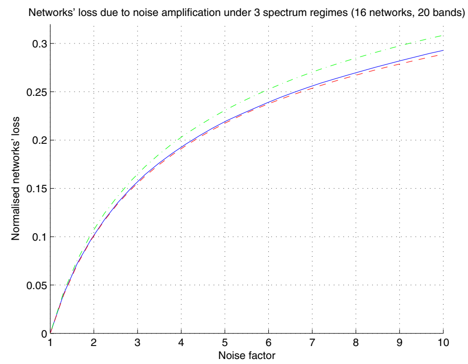
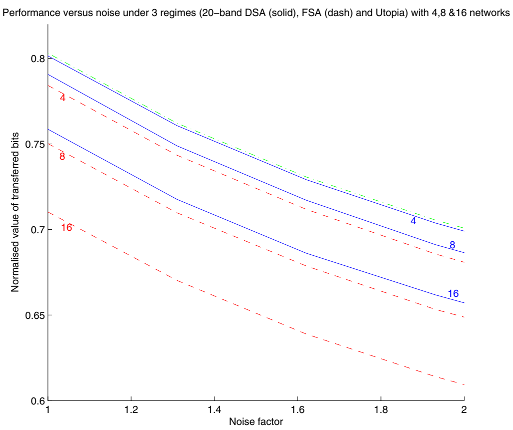


Figure 4: With a symmetry index of a ($0 < a \leq 1$), the market share of network $j+1$ is a times that of network j (e.g., with $a = 1/2$, arrival rates are $\propto$ 1, 1/2 , 1/4, etc.). The greater the asymmetry, the greater the gain of 20-band DSA (solid line) over egalitarian FSA. DSA also outperforms market-share-FSA (dash line). Utopia (not shown) performs only marginally better than DSA.

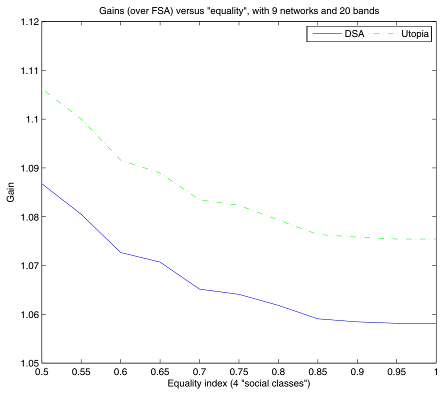


(a) Networks' loss is the difference between the original value (nf=1), and the current value (nf>1). It initially grows linearly with noise, but slows down around 3. All regimes are affected roughly equal.

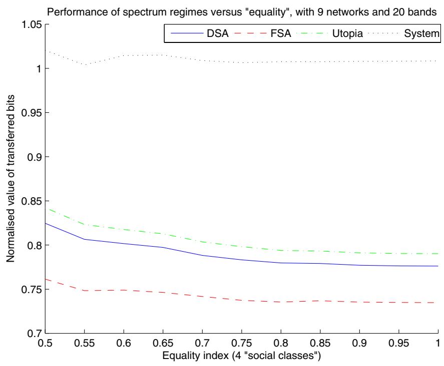


(b) With total spectrum and market size held constant, the fewer the networks, the better the performance. An increase in the number of network hurts more under FSA. With 4 networks, DSA approaches utopian performance.

Figure 5: Noise is amplified by the factor shown. Solid, dash and dash-dot lines represent normalised transferred-bit values under, respectively, DSA, FSA, and Utopia.



(a) DSA and utopian gains grow with inequality



(b) The greater the inequality (lower a), the better the performance for all regimes (in absolute terms).

Figure 6: The 4 equally-likely wtp values ("social classes") are the components of $4v/s$ where v=[1 a a^2 a^3] and s the sum of v's components (e.g., with a=1/2, v=[1 1/2 1/4 1/8] and s=15/8). The expected value of $4v/s$ is 1, for all a, so that the value of total system bits (dotted line in sub-fig. 6(b)) remains roughly constant.

in the downlink. The economic value of total transferred bits measures performance. The utopian regime is the clear winner, but DSA often finishes extremely close. DSA's *minimal* registered "gain" is 2-3%, where *identical* networks pool spectrum and customers. As new networks join this situation (for fixed spectrum and market size) the system performance suffers, but the DSA gain grows steadily with the number of networks and spectrum bands to 10% and beyond (thus, band size should be as small as technologically feasible). With 5 unevenly loaded networks, DSA outperforms egalitarian FSA by nearly 15%, and still significantly outperforms market-share-proportional FSA. The DSA and utopian gains also grow with "social inequality" (assessed through the wtp values) to around 10%. Notice that even a modest gain of a few percentage points at *every* DSA period can add up to an enormous sum throughout system lifetime, and can be the difference between financial success and failure.

Motivated by UWB, we consider a system-wide amplification of noise, and showed, for the 3 regimes, the resulting revenue loss to the networks. This loss is of the order of 10% of the total system value, when noise doubles. Such a high increase in noise is inconsistent with existing regulations, but is plausible, if (as advocated in [4]) more powerful UWB devices are allowed in exchange for "economic mitigation" from the beneficiaries to injured parties.

Acknowledgement

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