

Prioritized Throughput Maximization via Rate and Power Control for 3G CDMA: The 2 Terminal Scenario*

Virgilio Rodriguez
virgilio@poly.edu
ECE Department
Polytechnic University
Brooklyn, NY 11201

David J. Goodman
dgoodman@poly.edu
ECE Department
Polytechnic University
Brooklyn, NY 11201

October 04, 2002

Abstract

Variable spreading gain (VSG) CDMA allows data terminals to operate at dissimilar transmission rates. With this technique, the chip rate is common to all users, but the spreading gains vary. Our work is relevant to the uplink of a single-cell VSG CDMA system, in which each terminal's data throughput is weighted differently in calculating the network throughput. We seek for each active user a power level and transmission rate which will maximize the network's aggregate weighted throughput. This paper focuses on the two-terminal, interference-limited scenario. The development is entirely analytical, based on optimization theory. One of the principal results is that the favorite terminal should always operate at the highest data rate. The optimal bit rate of the other terminal depends on the ratio of the minimum allowable spreading gain to the square root of the priority coefficient. Only when this ratio is large enough, is it optimal for both terminals to operate at the highest permissible data rate. In either case, we provide equations whose solutions lead to the optimal power ratios.

1 Introduction

Modern wireless networks will accommodate simultaneous transceivers operating at very different bit rates. Several technologies have been proposed to accommodate multi-rate traffic in such networks. Ottosson and Svensson [4] discuss several multi-rate schemes based on Direct Sequence Code-Division Multiple Access (DS-CDMA). One such scheme is "variable spreading gain" (VSG) CDMA, as described, for example, by I and Sabnani[1]. In a VSG-CDMA system, the chip rate is constant across terminals, but the spreading gains may be different.

*Supported in part by the NSF through the grant "Multimodal Collaboration Across Wired and Wireless Networks", and by the NY State OSTAR through the Wireless Internet CAT.

Each terminal's spreading (processing) gain is determined as the ratio of the common chip rate to the terminal's bit rate.

The model discussed in this paper is relevant to an interference-limited single-cell VSG-CDMA system in which each data terminal can operate within a range of bit rates, which is assumed continuous for tractability. We seek an allocation specifying, for each active terminal, a choice of data rate and power level which will maximize the network weighted throughput. The weights may reflect different levels of "importance" among the terminals. The traffic is assumed to be delay-tolerant ("best-effort").

Similar situations have been considered by the literature, although these models are significantly different from the one considered herein. Our formulation has much in common with that of Ulukus and Greenstein [7]. Major differences between ours and their work include (a) our consideration of users' priorities, (b) our adoption of a generalized "frame-success" function (the function yielding the probability that a data packet is received successfully in terms of the terminal's received SIR), and (c) the simplifying linearization involved in their solution procedure. Our work has also many similarities with that of Sung and Wong [6]. They maximize a measure of channel "capacity" instead of throughput. But their use of a fairly general "capacity function" makes their work quite relevant to ours. On the other hand, they do not consider priorities, and, perhaps more significantly, assume that the terminal's data rates are fixed exogenous parameters, as opposed to variables to be chosen optimally.

Other related works seek decentralized solutions. Kim, et. al. [2] is noteworthy in that they assume integer data rates. This assumption makes their model more realistic, but limits considerably the analytical techniques available for a solution. They formulate rate/power control within a linear model, whose solution can be attained through either of two proposed distributed algorithms. This model assumes specific received SIR requirements for each considered data rate.

This paper examines a situation in which one base station receives CDMA signals from two data terminals. This study provides the core of a more general analysis.

The optimization analysis first seeks an allocation satisfying the first-order necessary optimizing conditions (FONOC), in which both spreading gains are strictly greater than their minimum permitted value. One such allocation is found and described by a closed-form expression. However, an analysis of the second-order sufficient optimizing conditions reveals that this allocation is neither a maximizer nor a minimizer, but a "saddle point". Thus, the maximization of the network's weighted throughput necessitates that at least one of the terminals operate at the smallest permissible spreading gain, G_0 , (which corresponds to the fastest permitted data rate). However, the saddle point is shown to have the interesting property of being "balanced", in the sense that both terminals enjoy the same weighted throughput.

Next, we seek an allocation satisfying the FONOC in which the spreading gain of the favorite (high priority) terminal is set to the smallest permitted value, G_0 (i.e. the favorite terminal operates at the fastest permitted data rate). The analysis shows that in order for such allocation to exist, an equation of the form $x^2 f'(x)/f'(\gamma_0) = G_0^2/\beta$ must have a solution. In this equation, f is the frame-success function mentioned above, $\beta \geq 1$ is the weight of the favorite terminal's throughput, and γ_0 is a specific SIR value clearly identified by the analysis. But the left hand side of this equation is bounded, and entirely determined by the physical layer through the frame-success function, f . Thus, if the ratio G_0^2/β exceeds this channel-determined bound, the concerned equation has no solutions. Therefore, when $G_0/\sqrt{\beta}$ is "large enough", the network maximizer involves both terminals operating at the fastest permissible data rate.

The "greedy" allocation in which both terminals are set to operate at the highest permissible data rate, with the power ratios determined through FONOC, is also investigated in the paper. A

detailed, closed-form solution is given for this situation, in the special symmetric case in which both terminals are equally “important”. It is shown that this “greedy” allocation is particularly treacherous, since it can lead to either a maximum or a minimum, depending upon whether the minimum permissible spreading gain, G_0 , exceeds a specific value determined by the physical layer through the frame-success function.

Below, we first build a relatively simple analytical model relevant to the uplink data transmission in one VSG-CDMA cell, and specify the general characteristics of the “frame-success” function giving the probability that a data packet is received successfully in terms of the terminal’s received SIR. Because this function is at the core of the analysis, it is desirable to impose as few restrictions as reasonable on it. Essentially, we just assume that the graph of this function is a smooth S-shaped curve. This characterization should accommodate a wide variety of coding and modulation schemes. Subsequently, we undertake the optimization problem of determining a rate and power choice for each of two terminals so that network’s weighted throughput is maximized. Finally, we provide further comments interpreting these results, and mention briefly relevant concurrent and future work.

2 Throughput Optimization

2.1 Problem Formulation

We will discuss a two-terminal situation, in which, for practical purposes, the noise power is negligible (the system is interference limited). We seek to solve:

$$\text{Maximize } \frac{f(G_1\alpha_1)}{G_1} + \frac{\beta f(G_2\alpha_2)}{G_2} \quad (1)$$

$$\text{subject to } \alpha_1\alpha_2 = 1 ; G_1 \geq G_0 ; G_2 \geq G_0$$

In this simple model,

1. $G_i = R_C/R_i$, $i \in \{1, 2\}$ is the spreading gain of terminal i ; i.e., the ratio of to the channel’s chip rate , R_C to its data rate R_i (bits per second). $G_0 \geq 1$ is the minimum permitted spreading gain (determined by the maximum permissible transmission rate).
2. α_i is the carrier-to-interference ratio (CIR) of the signal from terminal i received at the base station. α_i is defined as,

$$\alpha_i := \frac{P_i h_i}{\sum_{\substack{j=1 \\ j \neq i}}^N P_j h_j + \sigma^2} = \frac{Q_i}{\sum_{\substack{j=1 \\ j \neq i}}^N Q_j + \sigma^2}$$

with N the number of active terminals, P_i the transmission power of terminal i , h_i its “gain” (path loss) coefficient, $h_i P_i := Q_i$ its received power, and σ^2 a representative of the average noise power. With $N = 2$, assuming that $\sigma^2 = 0$ leads immediately to the constraint $\alpha_1\alpha_2 = 1$ ($\alpha_1 := Q_1/Q_2 := 1/\alpha_2$)

3. The product $G_i\alpha_i$, denoted as γ_i , is terminal i ’s signal to interference (SIR) ratio.
4. $\beta \geq 1$ is a priority/importance coefficient

5. We assume that there is a real-valued “frame-success” function which gives the probability of the correct reception of a packet in terms of the transmitter’s received signal-to-interference ratio. We assume that this function is such that $f(x) := f_S(x) - f_S(0)$ has the general properties of the generalized “sigmoidal” function (generic S-curve) discussed in [5], and that it has a continuous second derivative. The difference between f_S and f is generally negligible for practical purposes. Nevertheless, certain expressions involving f technically behave better.

2.2 Augmented objective function

In order to obtain the first order necessary optimizing conditions (FONOC) for an extremum, the following “extended” objective function is constructed:

$$\phi(G_1, G_2, \alpha_1, \alpha_2) = \frac{f(G_1\alpha_1)}{G_1} + \frac{\beta f(G_2\alpha_2)}{G_2} + \lambda(\alpha_1\alpha_2 - 1) + \mu_1(G_0 - G_1) + \mu_2(G_0 - G_2) \quad (2)$$

2.3 General First-Order Necessary Optimizing Conditions (FONOC)

The general FONOC can be expressed in vector form, with $\gamma_i = G_i\alpha_i$, as:

$$\begin{bmatrix} (\gamma_1 f'(\gamma_1) - f(\gamma_1)) / G_1^2 - \mu_1 \\ \beta (\gamma_2 f'(\gamma_2) - f(\gamma_2)) / G_2^2 - \mu_2 \\ f'(\gamma_1) + \lambda \alpha_2 \\ \beta f'(\gamma_2) + \lambda \alpha_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

$$\text{with } \begin{cases} \alpha_1 \alpha_2 = 1 \\ \mu_1 (G_0 - G_1) = 0 \\ \mu_2 (G_0 - G_2) = 0 \end{cases} \quad (4)$$

In order to check the second-order sufficient conditions one needs the Hessian matrix of second partial derivatives of our augmented objective function, denoted as ϕ_{xx} .

$$\phi_{xx} = \begin{bmatrix} \psi(\gamma_1) & 0 & \alpha_1 f''(\gamma_1) & 0 \\ 0 & \beta \psi(\gamma_2) & 0 & \beta \alpha_2 f''(\gamma_2) \\ \alpha_1 f''(\gamma_1) & 0 & G_1 f''(\gamma_1) & \lambda \\ 0 & \beta \alpha_2 f''(\gamma_2) & \lambda & \beta G_2 f''(\gamma_2) \end{bmatrix} \quad (5)$$

Where, for notational convenience, we define :

$$\psi(\gamma_i) = \frac{2}{G_i^3} \left[\frac{1}{2} \gamma_i^2 f''(\gamma_i) - \gamma_i f'(\gamma_i) + f(\gamma_i) \right] \quad (6)$$

2.4 Interior (‘balanced’) solution

First we shall seek an interior stationary point. That is, we presume that a solution to the first-order conditions exist in which both G_1 and G_2 are greater than G_0 , which require $\mu_1 = \mu_2 = 0$ (see equations (4)). Then, we proceed to check whether a solution to equations (3) and (4) consistent with these hypotheses actually exists.

2.4.1 Solving the FONOC

Working with the top 2 rows of the matrix equation (3), we obtain:

$$\gamma_1 f'(\gamma_1) = f(\gamma_1) \quad \text{and} \quad \gamma_2 f'(\gamma_2) = f(\gamma_2)$$

These equations have the form:

$$x f'(x) = f(x) \quad (7)$$

Rodriguez[5] shows that for the class of sigmoidal functions f being considered, there is a unique positive value γ_0 which satisfies equation (7). This value can be graphically identified in the attached figure as the abscissa of the point where the graph of f is tangent to a ray emanating from the origin; that is, tangent to the straight line $y = f'(\gamma_0)x$.

Therefore, if any values of the variables of interest satisfy, under the stated hypotheses, equations (3) and (4), they must be such that:

$$G_1^* \alpha_1^* = G_2^* \alpha_2^* = \gamma_0 \quad (8)$$

It is also established by working with the next two rows of the matrix equation (3) that:

$$-\lambda = \frac{f'(G_1^* \alpha_1^*)}{\alpha_2^*} = \frac{\beta f'(G_2^* \alpha_2^*)}{\alpha_1^*} \quad (9)$$

Now, substituting equation (8) into equation (9), we obtain :

$$\frac{f'(\gamma_0)}{\alpha_2^*} = \frac{\beta f'(\gamma_0)}{\alpha_1^*} \Rightarrow \frac{\alpha_1^*}{\alpha_2^*} = \beta \Rightarrow \alpha_2^* = \frac{1}{\sqrt{\beta}} = \frac{1}{\alpha_1^*}$$

We now have found a complete ‘‘interior’’ solution to the first-order optimizing conditions:

$$\alpha_1^* = \frac{1}{\alpha_2^*} = \sqrt{\beta} \quad ; \quad G_1^* = \frac{\gamma_0}{\alpha_1^*} = \frac{\gamma_0}{\sqrt{\beta}} \quad ; \quad G_2^* = \frac{\gamma_0}{\alpha_2^*} = \sqrt{\beta} \gamma_0 \quad (10)$$

Notice that, in order for these values to be consistent with our original hypotheses, $G_i^* > G_0$; i.e., $G_0 \sqrt{\beta} < \gamma_0$.

Substituting the above values into the objective function yields

$$T_B = \frac{f(\gamma_0)}{G_1^*} + \frac{\beta f(\gamma_0)}{G_2^*} = \frac{f(\gamma_0) \sqrt{\beta}}{\gamma_0} + \frac{\beta f(\gamma_0)}{\gamma_0 \sqrt{\beta}} = 2 \frac{f(\gamma_0) \sqrt{\beta}}{\gamma_0} \quad (11)$$

Notice that this is a closed form solution. If the function f is known, γ_0 can be easily obtained graphically (see attached figure) or equation(7) can be solved numerically. For instance, under suitable assumptions, the frame-success function corresponding to non-coherent FSK with packet size $M=80$ is:

$$f(x) = \left[1 - \frac{1}{2} \exp\left(-\frac{x}{2}\right) \right]^{80} \quad (12)$$

In this case, $\gamma_0 = 10.75$ and T_B reduces to about $0.15 \sqrt{\beta}$.

This operating point has an interesting property: it is ‘balanced’ in the sense that both users experience the same weighted throughput: $f(\gamma_0) \sqrt{\beta} / \gamma_0$. The ‘fairness’ of this operating point may be a desirable feature in certain situations.

2.4.2 Second-order sufficient conditions

The previously found allocation does satisfy the first-order necessary conditions for an optimizer (it is a “stationary” point). But we do not yet know whether it is actually a maximizer. As it turns out, it is not. It can be shown that it is a saddle point.

The optimality of this stationary point depends upon the matrix of second partial derivatives (Hessian matrix) of ϕ , our augmented objective function, which we denote as ϕ_{xx} . Essentially, at a point satisfying the FONOC, for any vector \vec{h} along a feasible direction, the triple product $\vec{h}^T[\phi_{xx}]\vec{h}$ is positive if the stationary point corresponds to a local minimum, and this product is negative if the stationary point corresponds to a local maximum. If neither of these conditions hold, then the point is a “saddle point”.

A feasible direction is one that is tangent to the curve representing the constraint relationship. Hence, if we denote our constraint curve as $b(G_1, G_2, \alpha_1, \alpha_2) := \alpha_1\alpha_2 - 1 = 0$, we only need to consider vectors \vec{h} satisfying $\nabla\mathbf{b} \bullet \vec{h} = 0$, that is, vectors normal to the gradient of the constraint curve. For us, at the interior stationary point,

$$\nabla\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \alpha_2^* \\ \alpha_1^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1/\sqrt{\beta} \\ \sqrt{\beta} \end{bmatrix} \propto \begin{bmatrix} 0 \\ 0 \\ 1 \\ \beta \end{bmatrix}$$

Then, it is easily verified that any vector \vec{h} of the form $[a_1 \ a_2 \ \beta a_3 \ -a_3]^T$, where the a_i 's are non-negative real numbers, satisfies $\nabla\mathbf{b} \bullet \vec{h} = 0$. It will prove convenient to express such vector as the product of a “transformation” matrix, \mathbf{M} , by an arbitrary vector $\vec{a} \doteq [a_1 \ a_2 \ a_3]^T$. It is trivial to verify that

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \beta \\ 0 & 0 & -1 \end{bmatrix}$$

is such that $\vec{h} = \mathbf{M} \cdot \vec{a}$ satisfies the desired condition.

The second-order conditions for the stationary point under consideration can be expressed in terms of the matrix $\mathbf{M}^T\phi_{xx}\mathbf{M}$. This matrix is positive definite if the stationary point corresponds to a local minimum, and is negative definite if the stationary point corresponds to a local maximum. This matrix is indefinite if this point is a “saddle point”. A square matrix is positive definite if all its principal minor determinants are positive.

ϕ_{xx} is given in equation (5).

But at our stationary point, (see equation (10)), ϕ_{xx} becomes, with $r_0 := f'(\gamma_0)/f''(\gamma_0)$:

$$\phi_{xx} = \begin{bmatrix} \beta/\gamma_0 & 0 & 1 & 0 \\ 0 & 1/\gamma_0\beta & 0 & 1 \\ 1 & 0 & \gamma_0/\beta & -r_0 \\ 0 & 1 & -r_0 & \beta\gamma_0 \end{bmatrix} \sqrt{\beta}f''(\gamma_0)$$

After some algebra, we obtain:

$$\frac{\mathbf{M}^T \times \phi_{xx} \times \mathbf{M}}{\sqrt{\beta}f''(\gamma_0)} = \begin{bmatrix} \frac{\beta}{\gamma_0} & 0 & \beta \\ 0 & \frac{1}{\gamma_0\beta} & -1 \\ \beta & -1 & 2\beta(\gamma_0 + r_0) \end{bmatrix} \quad (13)$$

The first principal minor determinant is simply the first element of the matrix, which clearly is a positive number. The second principal minor determinant is simply $1/\gamma_0^2$, which is also clearly positive. However, after some algebra we obtain that the determinant of the whole matrix is $2\beta r_0/\gamma_0^2 = 2\beta f'(\gamma_0)/\gamma_0^2 f''(\gamma_0)$.

But this expression is negative, because the first derivative of f is positive everywhere, and it can be shown (see [5]) that, for the class of functions f being considered, $f''(\gamma_0)$ is negative. Hence, we have the first two principal minor determinants positive, but the third one negative. The concerned matrix is indefinite. Therefore, our interior stationary point is neither a minimizer nor a maximizer. It is a saddle point.

2.5 Asymmetric-rates boundary solution

In the preceding section, we identified a solution to the FONOC lying in the interior of the feasible region (both spreading gains are greater than the minimum permissible value). This allocation was shown to be a non-maximizer, which suggests that the maximizer be sought over the “boundary” of the feasible region. An intuitively appealing boundary solution is that in which the “favorite” terminal transmits at the highest allowable bit rate, which means its spreading gain is G_0 ($\mu_2 \neq 0$), while G_1 is presumed inside its allowable range ($\mu_1 = 0$).

2.5.1 Solving the FONOC

Working with the first row of equation (3), and keeping in mind that we have presumed that $\mu_1 = 0$, we obtain $G_1\alpha_1 = \gamma_0$, with γ_0 as defined by equation (7), and shown in the attached figure.

Working with the last two rows of equation (3) we establish that:

$$-\lambda = \frac{f'(G_1\alpha_1)}{\alpha_2} = \frac{\beta f'(G_0\alpha_2)}{\alpha_1}$$

which can be re-written as follows:

$$\frac{G_0^2 f'(\gamma_0)}{G_0\alpha_2} = \beta f'(G_0\alpha_2) G_0\alpha_2 \Rightarrow \frac{x^2 f'(x)}{f'(\gamma_0)} = \frac{G_0^2}{\beta} \quad (14)$$

with $x := G_0\alpha_2$. Hence, the stationary value of α_2 is obtained by solving an equation of the form $x^2 f'(x) = K_0$, with $K_0 = f'(\gamma_0) G_0^2/\beta$, a system-dependent constant.

For the class of functions being considered, $x^2 f'(x)$ is a “bell-shaped” function defined over the non-negative side of the real line, as shown by the attached figure, and so is the function $x^2 f'(x)/f'(\gamma_0)$, since $f'(\gamma_0)$ is a positive constant.

This implies that, if G_0^2/β is too large, it may surpass the “peak” of the function on the left hand side of equation (14). Thus, this equation may have *no* solutions.

On the other hand, if G_0^2/β is sufficiently small, two values of x will satisfy equation(14). The larger value, to the right of “the peak”, is chosen as a prospective maximizer. Let γ_{00} be the largest value, if any, satisfying:

$$\frac{\gamma_{00}^2 f'(\gamma_{00})}{f'(\gamma_0)} = \frac{G_0^2}{\beta} \quad (15)$$

In terms of γ_{00} we can identify a complete solution. By definition, $\gamma_{00} = G_0\alpha_2$, which implies that $\alpha_2^* = \gamma_{00}/G_0$ satisfies the FONOC, and obviously so does $\alpha_1^* = 1/\alpha_2^* = G_0/\gamma_{00}$. And since the FONOC requires that $G_1^*\alpha_1^* = \gamma_0$, then G_1^* can be obtained as γ_0/α_1^* .

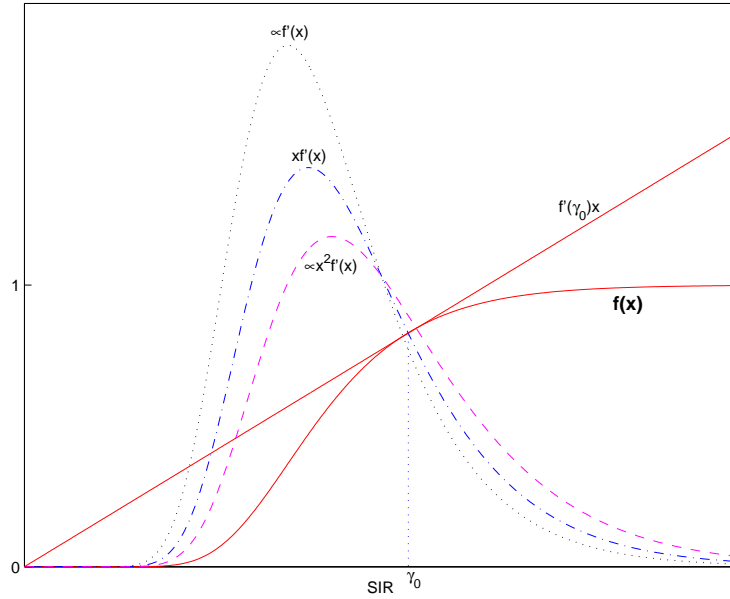


Figure 1: A particular $f(x)$, $xf'(x)$, and *scaled* versions of $f'(x)$, and $x^2f'(x)$. γ_0 satisfies $xf'(x) = f(x)$

Hence, we have arrived at the “asymmetric-rates” boundary allocation given by:

$$\begin{bmatrix} G_1^* \\ G_2^* \\ \alpha_1^* \\ \alpha_2^* \end{bmatrix} = \begin{bmatrix} \gamma_0 \gamma_{00} / G_0 \\ G_0 \\ G_0 / \gamma_{00} \\ \gamma_{00} / G_0 \end{bmatrix} \quad (16)$$

Notice that, for consistency with our hypothesis, G_1^* must be greater than G_0 , which requires that $G_0 < \sqrt{\gamma_0 \gamma_{00}}$.

Substituting these values into the objective function yields

$$T_{AR} = \frac{f(\gamma_0)}{G_1^*} + \frac{\beta f(\gamma_{00})}{G_2^*} = \frac{f(\gamma_0) G_0}{\gamma_{00} \gamma_0} + \frac{\beta f(\gamma_{00})}{G_0}$$

For instance, let us consider the frame success function introduced previously as equation (12).

We already know that for this function, $\gamma_0 = 10.75$, and $f(\gamma_0) = 0.83$.

When $G_0 = 2$ and $\beta = 2$, both $x = 22.1$ and $x = 3.97$ satisfy the equation $x^2 f'(x) = K_0$.

Hence, $\gamma_{00} = 22.1$. This gives $T_{AR} = 1.01$. By comparison, the ‘balanced’ solution only yields $T_B = 0.15\sqrt{2} = 0.21$, much less.

2.5.2 Second-order sufficient conditions

By applying a procedure similar to that of section (2.4.2) it can be verified that the allocation given by equation (16), if feasible, **is a maximizer**.

2.6 ‘Greedy’ allocation

In the preceding section, we considered the “asymmetric-rates” boundary solution, in which the “favorite” terminal operates at the smallest permissible spreading gain (fastest data rate),

while the spreading gain of the other terminal, as well as the power ratios, are determined by the analysis. It was observed that pre-setting the favorite terminal to operate at the highest permitted data rate will not lead to a point satisfying the first-order optimizing conditions if G_0^2/β is “too large”, and this allocation could be infeasible if $G_0 > \sqrt{\gamma_0\gamma_{00}}$. In this section we consider the ‘greedy’ situation, in which both terminals operate at the highest permissible data rate.

2.6.1 Solving the FONOC

Working with the last two rows of equation (3) we establish that:

$$-\lambda = \frac{f'(\gamma_1)}{\gamma_2} = \frac{\beta f'(\gamma_2)}{\gamma_1} \Rightarrow \gamma_1 f'(\gamma_1) = \beta \gamma_2 f'(\gamma_2) \quad (17)$$

with the constraint $\gamma_1 \gamma_2 = G_0^2$.

2.6.2 Second-order sufficient conditions

Proceeding as in section(2.4.2), we determine that a point satisfying equation (17) is a maximizer whenever $\gamma_1 h'(\gamma_1) + \beta \gamma_2 h'(\gamma_2) < 0$, where $h(x) := x f'(x)$ so that $h'(x) = f'(x) + x f''(x)$.

The symmetric case ($\beta = 1$).

To gain insight into the general case, we consider the special case in which $\beta=1$ (both terminals are equally “important”). In this case, it is a simple matter to verify that $\gamma_1 = \gamma_2 = G_0$ ($\alpha_1 = \alpha_2 = 1$) satisfies the FONOC. And the expression whose sign needs to be ascertained becomes $2G_0 h'(G_0)$. Hence, $\gamma_1 = \gamma_2 = G_0$ ($\alpha_1 = \alpha_2 = 1$) is a maximizer, whenever $h'(G_0) < 0$

This inequality is satisfied whenever G_0 is to the right of the point \hat{x} where $h(x) = x f'(x)$ reaches its maximum. This is so, because for the class of functions we are considering, the function $x f'(x)$ is a “bell-shaped” “single-peaked” curve extending between 0 and infinity (see attached figure). Hence, $h'(x) < 0$ for any point x , such that $x > \hat{x}$.

Accordingly, for $\beta = 1$, the point $\gamma_1 = \gamma_2 = G_0$ ($\alpha_1 = \alpha_2 = 1$) is a minimizer whenever $G_0 < \hat{x}$, and is a maximizer for $G_0 > \hat{x}$.

As an example, let us consider once again the frame-success function previously introduced as equation (12). In this case, $h(x) = x f'(x)$ reaches its maximum at $\hat{x} = 7.95$. Thus, in this particular case, for $\beta = 1$, $\gamma_1 = \gamma_2 = G_0$ ($\alpha_1 = \alpha_2 = 1$) is a minimizer for $G_0 < 7.95$ but is a maximizer for $G_0 > 7.95$.

3 Discussion

We have derived the optimum power levels and data rates for two terminals transmitting to one base station, in a scenario relevant to variable spreading gain CDMA. The objective function is the weighted network throughput, where the weights may indicate relative levels of importance (priorities) of the two terminals. The analysis leads to three power and rate assignments: a balanced assignment with both terminals operating within the range of permitted bit rates and achieving equal weighted throughput; an “unfair” assignment in which the favorite terminal operates at maximum bit rate, while the other terminal operates within the set of permitted rates; and a greedy assignment in which both terminals operate at maximal bit rate.

The balanced assignment is always suboptimal, which suggests that “fairness” (in the sense of equal weighted throughput) comes at the expense of performance. It is always optimal for the favorite terminal to operate at the maximum permissible data rate (data “speed limit”).

The network should admit both terminals at this maximal rate (“greedy” assignment optimal) when G_0 , the ratio of chip rate to the maximum bit rate, is greater than a “scaled” threshold, which is entirely determined by the details of the physical layer as embodied in the frame success function, f . The threshold’s scaling factor is $\sqrt{\beta}$, where β is the weight reflecting (possibly) the level of importance of the favorite terminal. This fact indicates that the chip rate necessary to optimally admit the low-priority terminal at maximum bit rate increases as the favorite terminal grows in importance. Alternatively, the data “speed limit” under which it is optimal to admit both terminals at maximum bit rate *decreases* when the favorite terminal’s importance grows. That is, the greater the importance of the favorite terminal, the slower the data “speed limit” which makes optimal for both terminals to operate at “full (data) speed”.

Relevant concurrent or future work by the authors includes consideration of non-negligible noise at the base station, and of an arbitrary number of terminals, as well as application of game theory to the derivation of decentralized control algorithms.

References

- [1] I, Chih-Lin and K.K. Sabnani, “Variable spreading gain CDMA with adaptive control for true packet switching wireless network”, IEEE ICC, pp: 725 -730 vol.2, 1995
- [2] Seong-Lyun Kim, Z. Rosberg, and J. Zander “Combined power control and transmission rate selection in cellular networks”, Vehicular Technology Conference, - Fall, pp: 1653 -1657, vol. 3, 1999
- [3] Oh, Seong-Jun, T.L. Olsen, and K.M. Wasserman, “Distributed power control and spreading gain allocation in CDMA data networks” IEEE INFOCOM, pp: 379 -385, vol.2, 2000
- [4] Ottosson, T. , and A. Svensson, “Multi-rate schemes in DS/CDMA systems”, IEEE Vehicular Tech. Conf. , pp: 1006 -1010, vol.2, 1995
- [5] Rodriguez, V., “Maximizing the Ratio of a Generalized Sigmoid to its Argument”, Technical Report, ECE Department, Polytechnic University, June, 2002 <http://wireless.poly.edu>
- [6] C. W. Sung and W. S. Wong, “Power Control and Rate Management for Wireless Multimedia CDMA Systems”, IEEE Trans. Commun., vol. 49, no. 7, pp. 1215-26, July 2002
- [7] Ulukus, S. and L.J. Greenstein “Throughput maximization in CDMA uplinks using adaptive spreading and power control” IEEE ISSSTA 2000, pp: 565 -569 vol.2