Power and Data Rate Assignment for Maximal Weighted Throughput in 3G CDMA: A Global Solution with Two Classes of Users

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***Abstract*— Relevant to the uplink of a VSG-CDMA system, a technique part of 3G standards, this work investigates power and data rate allocations that maximize the network *weighted* throughput. Each terminal has one of 2 possible weights, which admit various practical interpretations. Earlier works of ours tell us that at least one terminal should operate at the highest available data rate, and that terminals *not* operating at this rate should operate at the same signal-to-interference ratio (SIR). We have also learned that when only two terminals of dissimilar weights operate at maximal data rate, the optimal SIR values for these terminals is obtained as an intersection point between an "X-shaped" graph arising from optimality conditions, and a "U-shaped" graph arising from the feasibility condition on power ratios. In the present work, we introduce a general procedure to seek an allocation that is a global optimizer. Our analysis is based on classical optimization theory, and should accommodate a wide variety of physical layer configurations.**

I. Introduction

Modern wireless networks will accommodate simultaneous transceivers operating at very different bit rates. Variable spreading gain (VSG) CDMA is one of the technologies chosen to accommodate multi-rate traffic in such networks[2]. In a VSG-CDMA system, each terminal's spreading gain is the ratio of the common chip rate to the terminal's bit rate. Our model is relevant to an interference-limited single-cell VSG-CDMA system in which each data terminal can operate within a range of bit rates, assumed continuous for tractability. We seek allocations of data rate and power levels which will maximize the cell weighted throughput. There are two possible weights, which admit various interpretations, including levels of importance or priority, "utilities", or monetary prices. We assume the traffic to be delay-tolerant.

In previous works we have learned that in order for an allocation to be a candidate for maximizer, it must be such that (i) *at least one* terminal operates at the highest available data rate, and (ii) terminals *not* operating at this rate should experience a specific signal-to-interference ratio (SIR), γ_0 [7], [8]. However, many such allocations are possible, depending upon the number of terminals operating at the highest available data rate. A key question we need to answer is how many terminals should operate at the highest available data rate ("favored terminals"). Once this question is answered, we also need to specify the optimal CIR for these terminals, as well as the optimal data rate for the other ("non-favored") terminals (which will be operating at the preferred SIR value, γ_0). Below, we provide an analytical procedure to answer these questions.

Reference [1] represents a related strand of work of ours, in which noise (out of cell interference) is assumed non-negligible, the data rates are fixed and identical, and the optimal number of terminals is sought, along with the power allocation. Other authors have considered situations relevant to ours. Our present formulation has much in common with that in [10]. Major differences between ours and their work include (a) our consideration of weights (b) our adoption of a "generalized" frame-success function (discussed below), and (c) the simplifying linearization involved in their solution procedure. Reference [3] also seeks data rates and power allocation, and consider a "sigmoidal-like" frame-success function; but it focuses on the downlink, does not consider weights, and provides a sub-optimal algorithmic solution based on pricing. Reference [9], maximizes a fairly general "capacity function"; but does not consider weights, and assumes that the data rates are different but fixed. Other related works seek decentralized solutions.

At the core of our analysis is a frame-success function (FSF) that gives the probability that a data packet is received successfully in terms of the terminal's received signal-to-interference ratio (SIR). This function depends on many physical attributes of the system, such as the modulation technique, the forward error detection scheme, the nature of the channel, and properties of the receiver. We do *not* impose any particular algebraic functional form ("equation") on the FSF. Rather, we assume that *all that is known* about this function is that its graph is a smooth S-shaped curve, as displayed in fig. 1, and base our analysis on properties derived from this shape. Hence, our analysis should apply to many physical layer configurations of practical interest. Reference [4] discusses further this modeling approach.

Below, a relatively simple optimization model relevant to uplink data transmission in one VSG-CDMA cell is built. We present the first-order optimizing conditions for the dual class situation of interest, and focus on a specific scenario in which a few equally "important" terminals share a cell with many "ordinary" terminals. We presume that the system can accommodate all the important terminals at the highest available data rate. But it is not clear how many, if any, of the ordinary terminals should be set to operate at this high rate, in order to maximize the cell's weighted throughput. A general solution procedure for this scenario is given. Finally, we discuss our results, and comment on future relevant research.

II. General Formulation

We seek to solve:

$$\text{Maximize } \sum_{i=1}^{N} \beta_i T_i(G_i, \alpha_i) \tag{1}$$

subject to

$$\sum_{i=1}^{N} \frac{\alpha_i}{1+\alpha_i} = 1 \tag{2}$$

$$G_i \geq G_0 \quad i \in \{1, \cdots, N\} \tag{3}$$

In this simple model,

1) N is the number of terminals.
2) The throughput of terminal i is defined as $R_C T_i(G_i, \alpha_i)$, with
$$T_i(G_i, \alpha_i) := \frac{f(G_i \alpha_i)}{G_i} \tag{4}$$
3) $G_i = R_C / R_i$, $i \in \{1, \ldots, N\}$ is the spreading gain of terminal i; i.e., the ratio of the channel's chip rate , R_C to the terminal's data transmission rate R_i (bits per second). $G_0 \geq 1$ is the lowest available spreading gain (determined by the highest available data rate).
4) α_i is the carrier-to-interference ratio (CIR) of the signal from terminal i received at the base station. α_i is defined as,
$$\alpha_i := \frac{P_i h_i}{\sum_{\substack{j=1 \\ j \neq i}}^{N} P_j h_j + \sigma^2} = \frac{Q_i}{\sum_{\substack{j=1 \\ j \neq i}}^{N} Q_j + \sigma^2} \tag{5}$$
with P_i the transmission power of terminal i, h_i its the path gain coefficient, $h_i P_i := Q_i$ its received power, and σ^2 a representative of the average noise power and, possibly, out-of-cell interference. It can be shown that, with $\sigma^2 = 0$, the CIR's must be such that $\sum \alpha_i/(1+\alpha_i) = 1$ (constraint (2)) to ensure feasibility [5].
5) The product $G_i \alpha_i$, denoted as γ_i , is terminal i's signal to interference (SIR) ratio.
6) $\beta_i \geq 1$ is a weight, which admits various practical interpretations. With no loss of generality, we can always set $1 = \beta_1 \leq \cdots \leq \beta_N$. In this work, we consider the special case in which $1 = \beta_1 = \cdots = \beta_{N_1}$ and $\beta = \beta_{N_1+1} = \cdots = \beta_{N_1+N_2}$ with $N_1 + N_2 = N$. Thus, there are N_1 "light weight" terminals and N_2 "important" ones. However, in the development we often leave the weights expressed as $\beta_1, \cdots, \beta_N$ to show the patterns.
7) We assume that there is a frame-success function (FSF) which gives the probability of the correct reception of a data packet in terms of the received SIR. We assume that this function is such that $f(x) := f_S(x) - f_S(0)$ has the general properties of the generalized "S-curve" discussed in [6] (see fig. (1)), and that it has a continuous second derivative. Because $f_S(0)$ is very small, the difference between f_S and f is generally negligible. Nevertheless, this correction is made for technical reasons, on the basis of [4]. No actual function is used, except to provide numerical examples. Our analysis should apply to a wide variety of physical layer configurations, as long as they give rise to an FSF with an S-shaped graph.

Constraint (2) can be written as

$$\sum_{i=1}^{N} \frac{1}{1+\alpha_i} = N - 1 \tag{6}$$

In the development below, an asterisk used as a superscript on a variable denotes a specific value of the variable which satisfies certain optimality condition. We refer to terminals operating at maximal data rate as "favored" or "favorite", and call those terminals in the high-weight class "important", as opposed to "ordinary".

III. First-Order Necessary Optimizing Conditions

In [7], [8] we discuss the general technical procedure to solve a problem such as this. This procedure leads to the following first-order necessary optimizing conditions (FONOC), with $\gamma_i = G_i \alpha_i$

$$\begin{bmatrix} \beta_1 \partial T_1(G_1, \alpha_1)/\partial G_1 - \mu_1 \\ \vdots \\ \beta_N \partial T_N(G_N, \alpha_N)/\partial G_N - \mu_N \\ \beta_1 f'(\gamma_1) + \lambda(1+\alpha_1)^{-2} \\ \vdots \\ \beta_N f'(\gamma_N) + \lambda(1+\alpha_N)^{-2} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{7}$$

$$\text{with} \begin{cases} \sum_{i=1}^{N} (1+\alpha_i)^{-1} = N-1 \\ \mu_1 (G_0 - G_1) = 0 \\ \vdots \\ \mu_N (G_0 - G_N) = 0 \end{cases} \tag{8}$$

Notice that

$$\frac{\partial T_i(G_i, \alpha_i)}{\partial G_i} = \frac{\gamma_i f'(\gamma_i) - f(\gamma_i)}{G_i^2} \tag{9}$$

IV. Solving FONOC

A. Interior solution

It is natural to start by seeking an "interior" solution to FONOC, in which each G_i is greater than G_0, which requires $\mu_i = 0$, (see equations (8)). In [7] we show that, if G_0 is not "too large", one such solution exists and can be described by a closed-form expression. Unfortunately, this allocation is always a "saddle point" (neither a maximizer nor a minimizer).

B. Single-Favorite Boundary Solution (SFBS)

The fact that the interior solution to FONOC is neither a maximizer, nor a minimizer indicates that the true maximizer is a solution in which one or more terminals operate at the lowest available spreading gain. In [7], we start by seeking an allocation satisfying FONOC in which only the spreading gain of the "most important" terminal is set at the lowest available value, G_0 (i.e. this terminal operates at the highest available data rate), with other terminals' spreading gains to be determined by the analysis. Our analysis shows that in order for this single-favorite solution to exist, the SIR of the "non-favored" terminals should be obtained by solving an equation of the general form:

$$x f'(x) = f(x) \tag{10}$$

Reference [6] shows that if f is an S-curve, there is a unique positive value γ_0 which satisfies eq. (10). This value can be graphically identified in fig. 1 as the abscissa of the point where the graph of f is tangent to a ray emanating from the origin.

The SIR of the favorite terminal should be a solution to the equation

$$(C_1 x/G_0 + D_1)^2 f'(x)/f'(\gamma_0) = \frac{1}{\beta_N} \tag{11}$$

with

$$C_1 = \frac{N-1}{B_{N-1}} \; ; \; D_1 = \frac{N-2}{B_{N-1}} \tag{12}$$

$B_{N-1} := \sum_{j=1}^{N-1} \sqrt{\beta_j}$. In the special case in which there is only one important terminal, $\beta_N = \beta$, and $\beta_j = 1$ for $j = 1, \ldots, N-1$. Thus, $B_{N-1} := N-1$, $C_1 = 1$, and $D_1 = (N-2)/(N-1)$.

But the graph of the function $(C_1 x/G_0 + D_1)^2 f'(x)/f'(\gamma_0)$ has the same bell-shape of that of the function $x^2 f'(x)$ in fig. 1. Thus, the sought solution may not exist, because if G_0 is sufficiently large, the peak of this function may fall below $1/\beta$, unless β is also "very large".

Otherwise, two values of x on either side of the peak of the concerned function, say $x_1^* \leq x_2^*$, will satisfy equation (11). It is necessary for a maximum that $\mu_N \leq 0$, (μ_N is the Lagrange multiplier associated with the constraint $G_0 - G_N \leq 0$). After some analysis, this requirement implies that the chosen solution of eq. (11) must be greater than γ_0. This will generally rule out the smallest of the two solutions.

With x^* denoting the chosen of the two solutions to eq. (11), the FONOC-solving CIR for terminals 1 through N-1 is obtained from:

$$\alpha_i^* = \frac{1}{\sqrt{\beta_i}} \frac{B_{N-1}}{N-1-(1+x^*/G_0)^{-1}} - 1 \tag{13}$$

Since non-favored terminals operate at the SIR of γ_0, the matching spreading gain for a given α_i^* is $G_i^* = \gamma_0/\alpha_i^*$. In order for x^* to be useful, it must be such that $\gamma_0/\alpha_i^* > G_0$, the lowest available spreading gain.

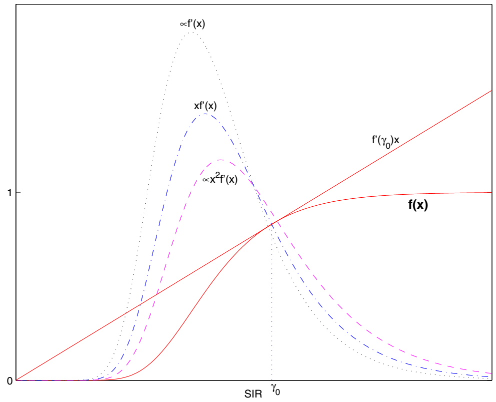

Fig. 1. A particular $f(x)$, $xf'(x)$, and *scaled* versions of $f'(x)$, and $x^2 f'(x)$. γ_0 satisfies $xf'(x) = f(x)$

C. A Multi-Favorite Boundary Solution (MFBS)

The single favorite boundary solution to FONOC may *not* exist, and even if it does exist, it may *not* lead to a global maximizer. We now investigate a more general solution to FONOC in which all the important terminals, N_2, and several ordinary terminals, say $n_1 \leq N_1$, operate at maximal data rate. Recall that $\beta_i = 1$ for $i = 1 \ldots N_1$, and $\beta_i = \beta > 1$ otherwise.

1) General structure of the solution: There are $N_1 - n_1$ non-favored terminals. Thus, $\mu_i = 0$ for $1 \leq i \leq N_1 - n_1$ (see equations (8)). Working with the first $N_1 - n_1$rows of the vector equation (7), we obtain, for $1 \leq i\ , j \leq N_1 - n_1$,

$$\gamma_i f'(\gamma_i) - f(\gamma_i) = 0 \rightarrow \gamma_i^* \equiv G_i^* \alpha_i^* = \gamma_0 \tag{14}$$

with γ_0 defined as the unique positive solution to eq. (10).

We also establish by working with the bottom half of the vector equation (7) that:

$$-\lambda = f'(G_i^* \alpha_i^*)\,(1+\alpha_i^*)^2 \;\text{ for } 1 \leq i \leq N_1 - n_1 \tag{15}$$

and

$$-\lambda = f'(G_0 \alpha_i^*)\,(1+\alpha_i^*)^2 \text{ for } N_1 - n_1 < i \leq N_1 \tag{16}$$

and

$$-\lambda = \beta f'(G_0 \alpha_i^*)\,(1+\alpha_i^*)^2 \text{ for } N - N_2 \leq i \leq N \tag{17}$$

Combining equations (14) and (15), we obtain

$$\alpha_i^* = \alpha_1^* \text{ for } 1 \leq i \leq N_1 - n_1 \tag{18}$$

For $N_1 - n_1 < i \leq N_1$, eq. (16) leads to n_1 equations of the form

$$f'(G_0 \alpha_i^*)\,(1+\alpha_i^*)^2 = f'(G_0 \alpha_j^*)\,\left(1+\alpha_j^*\right)^2$$

Evidently, this equation is satisfied with

$$\alpha_i^* = \alpha_j^* = y/G_0 \text{ for } N_1 - n_1 < i\ , j \leq N_1 \tag{19}$$

A similar analysis of eq. (17) leads to

$$\alpha_i^* = \alpha_j^* = x/G_0 \text{ for } N - n_2 < i\ , j \leq N \tag{20}$$

Now, the constraint relation (6) ($\sum_i (1+\alpha_i)^{-1} = N-1$) becomes an equation with only three unknowns, α_1, x and y. Substituting eqs. (18, 19 and 20) into (6) yields

$$\frac{N_1 - n_1}{1+\alpha_1} + \frac{n_1}{1+\frac{y}{G_0}} + \frac{N_2}{1+\frac{x}{G_0}} = N-1 \tag{21}$$

Equations (15, 16, and 17) imply that

$$f'(y)\left(1+\frac{y}{G_0}\right)^2 = \beta f'(x)\left(1+\frac{x}{G_0}\right)^2 \tag{22}$$

$$\beta f'(x)\left(1+\frac{x}{G_0}\right)^2 = f'(\gamma_0)\left(1+\alpha_1^*\right)^2 \tag{23}$$

Equation (23) provides a closed-form expression for α_1^* in terms of x:

$$\alpha_1^* = \left(1+\frac{x}{G_0}\right)\sqrt{\frac{\beta f'(x)}{f'(\gamma_0)}} - 1 \tag{24}$$

The function on the right-hand side of eq. (24) takes on values as low as -1, and yields a bell-shaped graph (such as that shown at the top of fig. 2). But, physically, α_1 cannot be negative. Thus, the existence of a MFBS in which all the ordinary terminals are active necessitates that the SIR of the favorite terminal be held within certain interval. This range expands as β grows, but shrinks as G_0 increases. Furthermore, α_1 cannot be too large, either. This is so because, in order to satisfy FONOC, the non-favored terminals must operate with SIR equal to γ_0. Thus the spreading gain for these terminals must equal γ_0/α_1. But if α_1 is large, this ratio may be smaller than G_0, which is the smallest allowable spreading gain. That is, it is necessary that $0 < \alpha_1 \leq \gamma_0/G_0$. This further constraint the values of x that can be chosen.

Within the appropriate range, eq. (24) allows us to write eq. (21) as :

$$\frac{n_1}{1+\frac{y}{G_0}} + \frac{N_1 - n_1}{\left(1+\frac{x}{G_0}\right)\sqrt{\frac{\beta f'(x)}{f'(\gamma_0)}}} + \frac{N_2}{1+\frac{x}{G_0}} = N-1 \tag{25}$$

Equations (22) and (25) form a system of two non-linear equations in two unknowns which is, in principle, solvable. Once we know the appropriate values of x^* and y^*, we can find α_1^*, the optimal CIR for terminals $1 \ldots N-n_1$, from eq. (24), and the corresponding spreading gain, from eq. (14), as γ_0/α_1^*. Thus, from x^* and y^* we can obtain a complete multi-favorite solution to FONOC. Below, we describe this solution, and comment on its optimality.

2) Discussion of the MFBS: The caption of figure 2 summarizes much of what can be said about the MFBS. Further insights are given in section V-B through numerical examples.

Generally, there are four intersection points, one in each of the "legs" of the concerned X-curve. It is clear that, among the four intersection points, the NE one yields the largest throughput for the favorite terminals, since both x (the SIR of the important terminals) and y (the SIR of the favored non-important terminals) are as high as possible. But if the number of non-favored terminals, $N_1 - n_1$, is larger than the number of favorites, $N_2 + n_1$, it is in principle possible that the

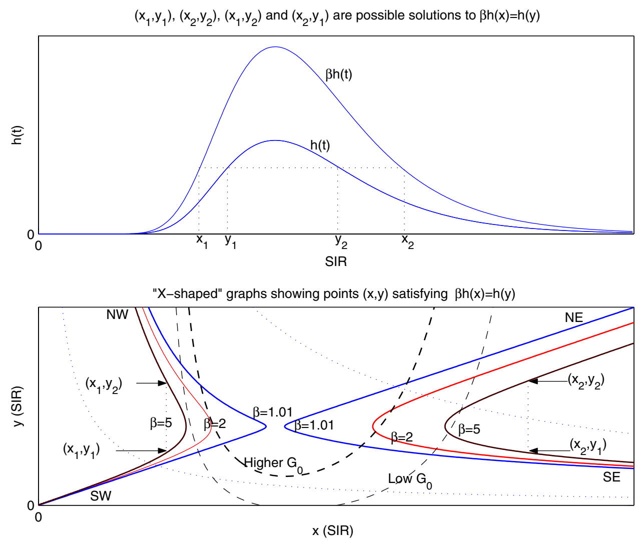

Fig. 2. With the SIR of the favored terminals denoted as x (important) and y (ordinary), FONOC requires that $\beta h(x) = h(y)$ (eq. (22)). Any of the pairs (x_1, y_1), (x_2, y_2), (x_1, y_2), or (x_2, y_1) (top) satisfies this equation, but may not be feasible. We plot all such points, which reveals an "X-shaped" graph (NE, NW, SW and SE are directional labels). On the same axes, we plot the U-shaped graph arising from the constraint equation (25). The 4 intersection points between the U-shaped and X-shaped graphs for the given (G_0,β) pair lead to feasible solutions to FONOC, provided that the resulting CIR and data rate for the non-favored terminals are also feasible. When *all* terminals operate at the highest data rate, a hyperbolic curve (from eq. (26)) replaces the U curve. If G_0 is low enough, the hyperbola may *only* intersect the SW leg of the X-curve, which leads to a minimum.

NE intersection point *not* lead to the largest overall weighted throughput. On the other hand, by considering the signs of the Lagrange multipliers associated with the spreading gains of the favored terminals, it is determined that both x and y must be greater than γ_0 for a maximum. This will typically rule out the intersection points falling outside the NE "leg" of the X.

Moreover, when we make $n_1 = N_1$ so that *all* terminals, whether important or not, operate at maximal data rate, then the U-curve is replaced by a hyperbolic "L-curve", as displayed in fig. 2. To see this more clearly, observe that when $n_1 = N_1$, we can solve eq. (25) for y in terms of x, obtaining:

$$\frac{y}{G_0} = \frac{N_1}{N_1 + N_2 - 1 - \frac{N_2}{1+x/G_0}} - 1 \tag{26}$$

For $x = 0$, $y = G_0/(N_1 - 1)$; and as $x \to \infty$, $y \to -G_0(N_2-1)/(N_1+N_2-1)$.Thus, when all terminals operate at maximal data rate, if $N_2 > 1$ (several "heavy-weight" terminals), there is an SIR value x beyond which y would have to be negative in order to satisfy the constraint on the power ratios, eq. (6). That is, the "L-curve" falls below zero for x sufficiently large. Hence, in this case, x cannot exceed $G_0/(N_2 - 1)$. Furthermore, for low G_0, the maximum value of y, which is $G_0/(N_1 - 1)$ could be so low, that the L-curve may intersect only the SW leg of the X-curve, in which case, both x and y are "low", and this would lead to a *minimum*, not a maximum. The message, in this case, is that there are too many "favored" terminals (those operating at the highest data rate); some need to be downgraded to "non-favored". On the other hand, with a large enough G_0, the hyperbola intersects the "Northern legs" of the X. In this case, a maximum results. Thus, when G_0 is large enough, all terminals should operate at the highest available data rate.

V. Finding the Global Maximizer

A. Solution procedure

In discussing the procedure, for expositional convenience we assume that there is only one important terminal ($N_2 = 1$). A key variable to be determined is the number of "favored" terminals (those operating at highest data rate). At least the important terminal must be in this group.

- Set $n_1 = 0$ (Single-favorite). Find, if possible, the 2 positive solutions to eq. (11), say x_1^* and x_2^* (if no such values exist, proceed to the next item). Either value can be a FONOC-solving SIR for the favorite terminal, and each leads to a complete allocation, but it is necessary that the chosen one be greater than γ_0 (section IV-B). For any of these values that is greater than γ_0, through eq. (13) obtain a corresponding α, the FONOC-solving CIR for the ordinary terminals, whose matching spreading gain is γ_0/α. If $\gamma_0/\alpha > G_0$, a complete *feasible* solution to FONOC has been found, and the corresponding weighted throughput can be calculated. This value may or may not be the global optimum. Set $n_1 = 1$ and proceed to find a dual-favorite solution.
- For $1 \leq n_1 < N_1$ (multifavorite solution) proceed as follows. Find the solutions (up to four) to the system of equations formed by eq. (22) and eq. (25). This is the equivalent of finding the four intersections between an X-shaped and a U-shaped graph (fig. 2). But not all of these intersections are useful. First, $\min\{x, y\} \geq \gamma_0$ for a maximum. Also, if the x value is outside certain range, the FONOC-solving CIR of the non-favored terminals, α, may be negative, or its matching spreading gain may be less than G_0. Each of the *useful* intersections determine a complete solution to FONOC. The SIRs of the favored terminals are x (important) and y (ordinary). The FONOC-solving CIR for the non-favored terminals can be found from eq. (24), and the matching spreading gain is γ_0/α. The corresponding weighted network throughput can then be calculated for each feasible solution. If the U curve is "too wide", meaning that x would make α negative, proceed to the next item, below. Otherwise, increment n_1 and repeat this complete item (draw another U curve for the new n_1), until $n_1 = N_1$.
- For $n_1 = N_1$ (all terminals, important or not, operate at the highest data rate), find the solution to the system of equations formed by eq. (22) and eq. (26). This is the equivalent of finding the intersections between an X-shaped graph and a hyperbola (fig. 2). The SIRs of the important terminal is x and that of the ordinary terminals is y. The matching CIRs are respectively x/G_0 and y/G_0. Each intersection leads to a feasible solution to FONOC, from which the weighted throughput can be calculated. $\min\{x, y\} \geq \gamma_0$ continues to be necessary for a maximum. If the only intersection lies in the SW leg of the X, the "all-favored" solution is a local minimizer (useless).
- The global maximizer is found among the feasible FONOC-solving allocations already discussed, and is whichever yields the largest weighted throughput.

B. Numerical examples

In the examples shown in figures 3 , 4 and 5, the frame-success function is $f(x) = [1 - (1/2)\exp(x/2)]^{80}$, corresponding to non-coherent FSK, no FEC, and packet size of 80 bits. The "preferred" SIR $\gamma_0 = 10.75$ for this FSF. There are 10 terminals, one of which is "important".

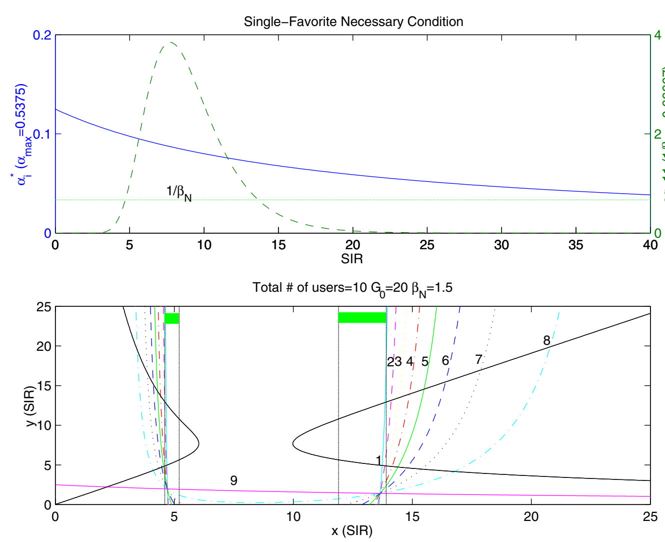

Fig. 3. With a moderate $G_0 = 20$ and $\beta = 1.5$, two single-favorite solutions exist, at $x \approx$4.5 and 13 (top). But the best yields only a throughput of 0.12 the chip rate. Fortunately, the bottom subplot shows 4 dual-favorite ($n_1 = 1$) solutions at (13.8, 12.8), (13.7, 4.9), (4.7,12.7) and (4.7,5.0) leading to *weighted* throughput of 0.7, 0.65, 0.05 and 0.015 the chip rate, respectively. The "all favored" solution ($n_1 = 9$) leads to a minimum.

The top subplot refers to the "single-favorite" solution (SFBS), with x the SIR of the favorite. The first order optimizing conditions (FONOC) require the SIR of the favorite to be at one of the intersections between the shown bell-shaped curve and the line $1/\beta$ (eq. (11)). The hyperbola at the top corresponds to α, the CIR of the non-favorites, as a function of x (eq. (13)). If α exceeds $\alpha_{\max} = \gamma_0/G_0$, its matching spreading gain γ_0/α is less than G_0, the lowest available. The bottom subplot corresponds to the multi-favorite solutions, in which the important terminal and n_1 ordinary ones operate with the lowest available spreading gain G_0, while the remaining $N_1 - n_1$ ordinary terminals operate with an SIR of γ_0. x and y are respectively the SIR of the important and ordinary terminals operating at the highest data rate ("favored"). The X-graph arises from eq. (22), and the U graph from eq. (25). U curves are numbered with the chosen n_1. The 4 intersection points between the U and X graphs may lead to feasible solutions to FONOC. But $\min\{x, y\} \geq \gamma_0$, and x must lie inside the intervals indicated by the thick green line. Outside these intervals, either the resulting CIR, α, for the non-favored terminals, or its matching spreading gain is unacceptable.

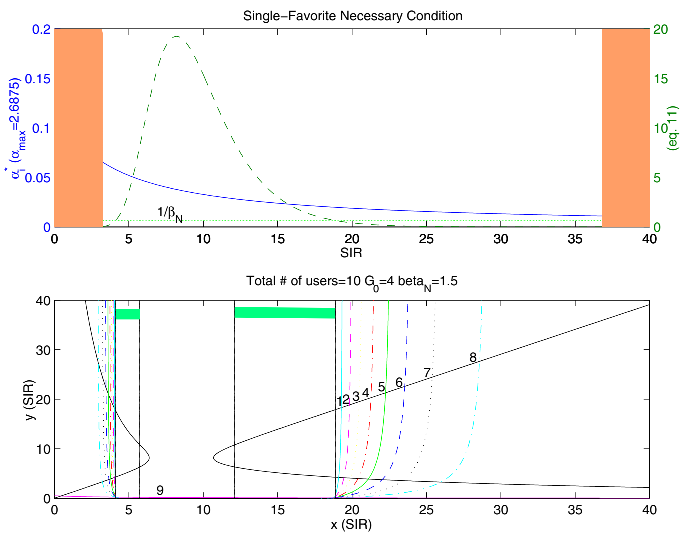

Fig. 4. With a small $G_0 = 4$ and a moderate $\beta = 1.5$ the single-favorite solution exists (top), and leads to the maximum. But all the multi-favorite solutions fail (intersections of U and X curves falls outside the acceptable range of x). An "all favored" solution exists (barely visible) but leads to a minimum.

VI. Discussion

We have investigated the optimal power levels and data rates for terminals transmitting to one base station, in a scenario relevant to 3G CDMA. The objective function is the *weighted* sum of each terminal's throughput. Two weights, which admit various interpretations, including levels of importance, "utilities", or monetary prices, are considered. The properties of the physical layer are embodied in the frame success function (FSF), which gives, in terms of received signal-to-interference ratio (SIR), the probability that a data packet is correctly received. But we do *not* impose any specific functional form ("equation") on the FSF. We assume that *all that is known* about the FSF is that its graph is "S-shaped", and base our analysis on properties derived from this shape. Therefore, we can accommodate many physical layer configurations of practical interest. Each physical layer has a preferred SIR, γ_0, easily identified in the graph of the FSF. We have presented a complete solution procedure, and provided and discussed numerical examples.

Our model could be expanded to consider the issues of QoS, fairness, and decentralized implementations, all of which are of practical importance. Introducing non-negligible noise, which we have started in [1], is also important, because the noise term may include out-of-cell interference.

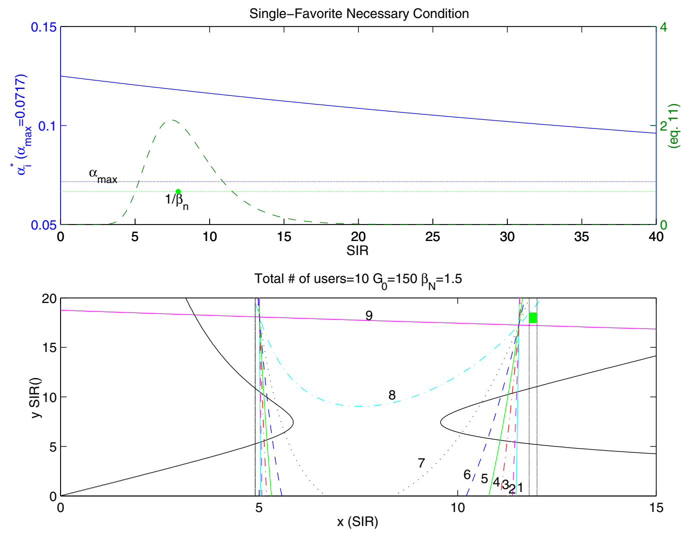

Fig. 5. With a high $G_0 = 150$ and $\beta = 1.5$ *no* single-favorite solution is available. Although the bell curve does intercept the line $1/\beta$ (top), for any x, α will be above 0.07, which means its matching spreading gain will fall below G_0. The same problem plagues multi-favorite solutions (U-X intersections) with $1 \leq n_1 \leq 8$, all of which falls outside the acceptable range for x (shown by the thick green lines). However, the "all-favored" solutions ($n_1 = 9$) (intersections of the hyperbola and the X) do exist. The NE intersection leads to the global maximizer.

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