Power and Data Rate Assignment for Maximal Weighted Throughput: A Dual-Class 3G CDMA Scenario

Virgilio Rodriguez, David J. Goodman
Polytechnic University
ECE Department
5 Metrotech Center
Brooklyn, NY 11201

Abstract— Relevant to the uplink of a VSG-CDMA system, a technique part of 3G standards, this work investigates power and data rate allocations that maximize the network *weighted* throughput. Each terminal has one of 2 possible weights, which admit various practical interpretations. Earlier works of ours tell us that at least one terminal should operate at the highest available data rate, and that terminals *not* operating at this rate should operate at the same signal-to-interference ratio (SIR). This value is determined by the physical layer through the function that gives, in terms of the received SIR, the probability that a data packet is received correctly. In this work, we provide a more specific description of the optimal allocations. The key to the solution is an intersection point between an "X-shaped" graph arising from optimality conditions, and a "U-shaped" graph arising from the feasibility condition on power ratios. Our analysis is based on classical optimization theory, and should accommodate a wide variety of physical layer configurations.

I. INTRODUCTION

Modern wireless networks will accommodate simultaneous transceivers operating at very different bit rates. Several technologies have been proposed to accommodate multi-rate traffic in such networks. Ottosson and Svensson [3] discuss several multi-rate schemes based on Direct Sequence Code-Division Multiple Access (DS-CDMA). One such scheme is variable spreading gain (VSG) CDMA, as described, for example, by I and Sabnani[1]. In a VSG-CDMA system, each terminal's spreading gain is defined as the ratio of the common chip rate to the terminal's bit rate.

Our model is relevant to an interference-limited single-cell VSG-CDMA system in which each data terminal can operate within a range of bit rates, assumed continuous for tractability. We seek allocations of data rate and power levels which will maximize the network weighted throughput. There are two possible weights, which admit various interpretations, including levels of importance or priority, "utilities", or monetary prices. We assume the traffic to be delay-tolerant.

Two recent publications of ours are relevant to this work. In [7], we study an identical situation, but we consider exclusively two terminals. In [8], we consider a multi-class scenario in which there is a distinct weight per user, and provide a general framework for this analysis, and a general

structure of solutions. However, at that level of generality, the answers to certain important questions are not available. In this work, we focus on a dual-class situation, which allows us to describe in greater detail the optimal allocations.

Other authors have considered situations relevant to ours. Our formulation has much in common with that of Ulukus and Greenstein [10]. Major differences between ours and their work include (a) our consideration of weights (b) our adoption of a "generalized" frame-success function (discussed below), and (c) the simplifying linearization involved in their solution procedure. A recent work by Lee, et al. [2] is also of interest. They also seek data rates and power allocation, and consider a "sigmoidal-like" frame-success function. But they focus on the downlink, do not consider weights, and provide a sub-optimal algorithmic solution based on pricing. Our work has also many similarities with that of Sung and Wong [9], who maximize a fairly general "capacity function". But they do not consider weights, and assume that the terminal's data rates are different but fixed. Other related works seek decentralized solutions.

At the core of our analysis is a frame-success function (FSF) that gives the probability that a data packet is received successfully in terms of the terminal's received signal-to-interference ratio (SIR). This function is determined by physical attributes of the system, such as the modulation technique, the forward error detection scheme, the nature of the channel, and properties of the receiver. We do *not* impose any particular algebraic functional form ("equation") on the FSF. Rather, we assume that all that is known about this function is that its graph is a smooth S-shaped curve, as displayed in fig. (1), and base our analysis on properties derived from this shape. Hence, our analysis should apply to many physical layer configurations of practical interest. Reference [4] discusses further this modeling approach.

Below, we first build a relatively simple optimization model relevant to uplink data transmission in one VSG-CDMA cell. Afterward, we provide an outline of the general solution procedure. Then, we apply this procedure to the dual class situation of interest. Finally, we discuss our results, and comment on future relevant research.

II. General Formulation

A. Problem Statement

We seek to solve:

$$\text{Maximize } \sum_{i=1}^{N} \beta_i T_i(G_i, \alpha_i) \tag{1}$$

subject to

$$\sum_{i=1}^{N} \frac{\alpha_i}{1 + \alpha_i} = 1 \tag{2}$$

$$G_i \geq G_0 \quad i \in \{1, \cdots, N\} \tag{3}$$

In this simple model,

1) N is the number of terminals.
2) The throughput of terminal i is defined as $R_C T_i(G_i, \alpha_i)$, with

$$T_i(G_i, \alpha_i) := \frac{f(G_i \alpha_i)}{G_i} \tag{4}$$

3) $G_i = R_C/R_i$, $i \in \{1, \ldots, N\}$ is the spreading gain of terminal i; i.e., the ratio of the channel's chip rate , R_C to the terminal's data transmission rate R_i (bits per second). $G_0 \geq 1$ is the lowest available spreading gain (determined by the highest available data rate).
4) α_i is the carrier-to-interference ratio (CIR) of the signal from terminal i received at the base station. α_i is defined as,

$$\alpha_i := \frac{P_i h_i}{\sum_{\substack{j=1 \\ j \neq i}}^{N} P_j h_j + \sigma^2} = \frac{Q_i}{\sum_{\substack{j=1 \\ j \neq i}}^{N} Q_j + \sigma^2} \tag{5}$$

with P_i the transmission power of terminal i, h_i its the path gain coefficient, $h_i P_i := Q_i$ its received power, and σ^2 a representative of the average noise power and, possibly, out-of-cell interference. It can be shown that, with $\sigma^2 = 0$, the CIR's must be such that $\sum \alpha_i/(1 + \alpha_i) = 1$ (constraint (2)) to ensure feasibility [5].

5) The product $G_i \alpha_i$, denoted as γ_i, is terminal i's signal to interference (SIR) ratio.
6) $\beta_i \geq 1$ is a weight, which admits various practical interpretations. With no loss of generality, we can always set $1 = \beta_1 \leq \cdots \leq \beta_N$. In this work, we consider the special case in which $1 = \beta_1 = \cdots = \beta_{N_1}$ and $\beta = \beta_{N_1+1} = \cdots = \beta_{N_1+N_2}$ with $N_1 + N_2 = N$. Thus, there are N_1 "light weight" terminals and N_2 "important" ones. However, in the development we often leave the weights expressed as $\beta_1, \cdots, \beta_N$ to show the patterns. Initially, we take $N_2 = 1$, but later we discuss a generalization.
7) We assume that there is a frame-success function (FSF), f_S, which gives the probability of the correct reception of a data packet in terms of the received SIR. We assume that this function is such that $f(x) := f_S(x) - f_S(0)$ has the general properties of the generalized "S-curve" introduced in [6] (see fig. (1)), and that it has a continuous second derivative. Because $f_S(0)$ is very small,

the difference between f_S and f is generally negligible. Nevertheless, we have good technical reasons to make this correction, on the basis of [4]. No actual function is used, except to provide numerical examples.

Constraint (2) can be written as

$$\sum_{i=1}^{N} \frac{1}{1 + \alpha_i} = N - 1 \tag{6}$$

In the development below, an asterisk used as a superscript on a variable denotes a specific value of the variable which satisfies certain optimality condition. We refer to terminals operating at maximal data rate as "favored" or "favorite", and call those terminals in the high-weight class "important", as opposed to "ordinary".

B. General solution procedure

The general procedure is as follows:
- Create an "augmented" objective function, combining the original objective function with Lagrange multipliers and the constraint equations
- Set up the first-order necessary optimizing conditions (FONOC). This involves setting the partial derivative of the *augmented* objective function with respect to each variable equal to zero. Moreover, inequalities of the form $G_0 - G_i \leq 0$ contribute equations of the form $\mu_i(G_0 - G_i) = 0$, where μ_i is a Lagrange multiplier.
- Solve FONOC. Evidently, each equation of the form $\mu_i(G_0 - G_i) = 0$ requires that if $G_i > G_0$, μ_i be equal to zero; and that if $\mu_i \neq 0$, G_i equal G_0. Both possibilities must be considered separately while finding various solutions to FONOC.
- A solution to FONOC provides a candidate for a maximizer. The second-order sufficient conditions *may* confirm the candidate as a maximizer. This maximizer may *not* be global.

III. Applying the General Solution Procedure

A. Augmented objective function

The "augmented" objective function is $\phi(G_1, \ldots, G_N, \alpha_1, \ldots, \alpha_N) =$

$$\sum_{i=1}^{N} \beta_i T_i(G_i, \alpha_i) + \lambda \left(\sum_{i=1}^{N} \frac{\alpha_i}{1 + \alpha_i} - 1 \right) + \sum_{i=1}^{N} \mu_i(G_0 - G_i) \tag{7}$$

B. General First-Order Necessary Optimizing Conditions (FONOC)

The general FONOC can be expressed in vector form, with $\gamma_i = G_i \alpha_i$, as:

$$\begin{bmatrix} \beta_1 \partial T_1(G_1, \alpha_1)/\partial G_1 - \mu_1 \\ \vdots \\ \beta_N \partial T_N(G_N, \alpha_N)/\partial G_N - \mu_N \\ \beta_1 f'(\gamma_1) + \lambda(1 + \alpha_1)^{-2} \\ \vdots \\ \beta_N f'(\gamma_N) + \lambda(1 + \alpha_N)^{-2} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{8}$$

$$\text{with} \begin{cases} \sum_{i=1}^{N}(1+\alpha_i)^{-1} = N-1 \\ \mu_1(G_0 - G_1) = 0 \\ \quad\vdots \\ \mu_N(G_0 - G_N) = 0 \end{cases} \tag{9}$$

Notice that

$$\frac{\partial T_i(G_i, \alpha_i)}{\partial G_i} = \frac{\gamma_i f'(\gamma_i) - f(\gamma_i)}{G_i^2} \tag{10}$$

C. Solving FONOC

1) Interior solution : It is natural to start by seeking an "interior" solution to FONOC, in which each G_i is greater than G_0, which requires $\mu_i = 0$, (see equations (9)). In [8] we discuss that, if G_0 is not "too large", one such solution exists and can be described by a closed-form expression.

In this solution, all terminals operate at an SIR value obtained by solving an equation of the general form:

$$x f'(x) = f(x) \tag{11}$$

Reference [6] shows that for the class of generalized sigmoidal functions, such as f, there is a unique positive value γ_0 which satisfies eq. (11). This value can be graphically identified in figure (1) as the abscissa of the point where the graph of f is tangent to a ray emanating from the origin; that is, tangent to the straight line $y = f'(\gamma_0)x$. Thus, if the function f is known, γ_0 can be easily obtained graphically or eq. (11) can be solved numerically. For instance, $\gamma_0 = 10.75$ and $f(\gamma_0) = 0.83$ for the FSF corresponding, under suitable assumptions, to non-coherent FSK with packet size M=80, which is

$$f_s(x) = \left[1 - \frac{1}{2}\exp\left(-\frac{x}{2}\right)\right]^{80} \tag{12}$$

However, the second-order optimality conditions reveal that this solution is always a "saddle point" (neither a maximizer nor a minimizer).

2) Single-Favorite Boundary Solution (SFBS): The fact that the interior solution to FONOC is neither a maximizer, nor a minimizer indicates that the true maximizer is a solution in which one or more terminals operate at the lowest available spreading gain. We start by seeking an allocation satisfying FONOC in which only the spreading gain of the "important" terminal is set at the lowest available value, G_0 (i.e. this terminal operates at the highest available data rate), with other terminals' spreading gains to be determined by the analysis.

In [8], we fully discuss this case, for the general situation in which there is a distinct weight for each terminal. Our analysis shows that in order for this single-favorite solution to exist, the SIR of the "non-favored" terminals should be the previously mentioned γ_0, and the SIR of the favorite terminal should be a solution to the equation $(C_1 x/G_0 + D_1)^2 f'(x)/f'(\gamma_0) = 1/\beta_N$. In this equation, C_1 and D_1 are constants which are largest when the weights of the non-favored terminals are smallest, and the other symbols are as previously defined. But the graph of the function $(C_1 x/G_0 + D_1)^2 f'(x)/f'(\gamma_0)$ has the same "bell-shape" of that of the function $x^2 f'(x)$ in fig. (1).

Thus, the sought solution may not exist, because if G_0 is "very large", the peak of this function may fall below $1/\beta_N$, unless β_N is also "very large". This makes intuitive sense, because when G_0 is "very large", the highest available data rate is relatively small, and keeping only one terminal operating at maximal data rate is not appealing, unless that terminal has "a lot of weight". However, when the highest available data rate is very high, an allocation in which *only one* terminal operates at this rate is more appealing.
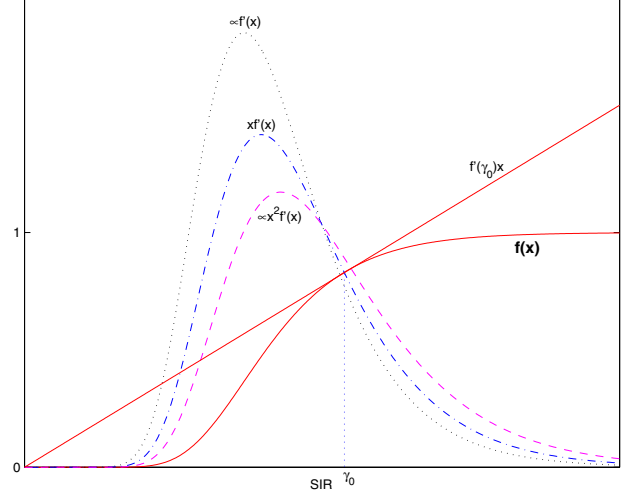

Fig. 1. A particular $f(x)$, $x f'(x)$, and *scaled* versions of $f'(x)$, and $x^2 f'(x)$. γ_0 satisfies $x f'(x) = f(x)$

3) Dual-Favorite Boundary Solution (DFBS) to FONOC: The "single favorite" boundary solution to FONOC may *not* exist, and even if it does exist, it may *not* lead to a global maximizer. We now seek an allocation satisfying FONOC, in which both the "important" terminal N, and the terminal whose index is $N-1$ ("sub-favorite") operate at the highest permissible data rate. Below, we set $G_N = G_{N-1} = G_0$, and $\mu_i = 0$ for $1 \le i \le N-2$, and seek a solution to FONOC satisfying these hypotheses.

a) General structure of the solution: Working with the first $N-2$ rows of the vector equation (8), and keeping in mind that our presumptions require that $\mu_i = 0$ for $1 \le i \le N-2$, we obtain

$$\gamma_i^* = G_i^* \alpha_i^* = \gamma_0 \text{ for } 1 \le i \le N-2 \tag{13}$$

with γ_0 defined as the unique positive solution to equation (11).

We also establish by working with the bottom half of the vector equation (8) that:

$$-\lambda = \beta_i f'(G_i^* \alpha_i^*)(1+\alpha_i^*)^2 \text{ for } 1 \le i \le N-2 \tag{14}$$

and

$$-\lambda = \beta_i f'(G_0 \alpha_i^*)(1+\alpha_i^*)^2 \text{ for } N-1 \le i \le N \tag{15}$$

Combining equations (13) and (14), and the fact that $\beta_i = 1$ for $1 \le i \le N-1$, we obtain

$$\alpha_i^* = \alpha_j^* \text{ for } 1 \le i, j \le N-2 \tag{16}$$

Now, the constraint relation (6) ($\sum_i (1 + \alpha_i)^{-1} = N - 1$) becomes an equation with only three unknowns, α_1, α_{N-1} and α_N. Substituting eq. (16) into (6) yields, with $x := G_0 \alpha_N^*$ and $y := G_0 \alpha_{N-1}^*$

$$\frac{N-2}{1+\alpha_1} + \frac{1}{1+\frac{y}{G_0}} + \frac{1}{1+\frac{x}{G_0}} = N - 1 \qquad (17)$$

Equations (14) and (15) imply that

$$f'(y)\left(1+\frac{y}{G_0}\right)^2 = \beta f'(x)\left(1+\frac{x}{G_0}\right)^2 \qquad (18)$$

$$\beta f'(x)\left(1+\frac{x}{G_0}\right)^2 = f'(\gamma_0)\left(1+\alpha_1^*\right)^2 \qquad (19)$$

Equation (19) provides a closed-form expression for α_1^* in terms of x:

$$\alpha_1^* = \left(1+\frac{x}{G_0}\right)\sqrt{\frac{\beta f'(x)}{f'(\gamma_0)}} - 1 \qquad (20)$$

The function on the right-hand side of eq. (20) takes on values as low as -1, and yields a bell-shaped graph (such as that shown at the top of fig. (2)). But, physically, α_1 cannot be negative. Thus, the existence of a DFBS in which all the ordinary terminals are active necessitates that the SIR of the favorite terminal be held within certain range. This range expands as β grows, but shrinks as G_0 increases. Within this range, eq. (20) allows us to write eq. (17) as :

$$\frac{1}{1+\frac{y}{G_0}} + \frac{N-2}{\left(1+\frac{x}{G_0}\right)\sqrt{\frac{\beta f'(x)}{f'(\gamma_0)}}} + \frac{1}{1+\frac{x}{G_0}} = N - 1 \qquad (21)$$

Equation (18) cannot be solved explicitly. However, equations (18) and (21) form a system of two non-linear equations in two unknowns which is, in principle, solvable. Once we know the appropriate values of x^* and y^*, we can find α_1^*, the optimal CIR for terminals $1 \ldots N - 2$, from eq. (20), and the corresponding spreading gain, from eq. (13), as γ_0/α_1^*. Thus, from x^* and y^* we can obtain a complete dual-favorite solution to FONOC. Below, we describe this solution, and comment on its optimality.

b) Description of the DFBS to FONOC: Figure (2) summarizes much of what can be said about the DFBS. For convenience, let $h(t) := f'(t)\left(1+t/G_0\right)^2$, with t a "dummy" variable. The SIR's of the two terminals operating at maximal bit rate, denoted as x and y, must satisfy eq. (18) in order to satisfy FONOC. For the class of functions being considered, the graph of $h(t)$ is "bell-shaped", as displayed in the top sub-figure of fig. (2). That is, there is exactly one point t^* at which this function has a global maximum, and for every $t_1 \leq t^*$ there is a $t_2 \geq t^*$ such that $h(t_1) = h(t_2)$. Thus, for every pair (x_2, y_2) which satisfies eq. (18), with $x_2 \geq t^*$ and $y_2 \geq t^*$, there is a corresponding pair (x_1, y_1), with $x_1 \leq t^*$ and $y_1 \leq t^*$, which also satisfies this equation, and so do (x_1, y_2), and (x_2, y_1).

For a given value of x, there are two values of y which satisfy $\beta h(x) = h(y)$, unless x is "too close" to t^*. The bottom
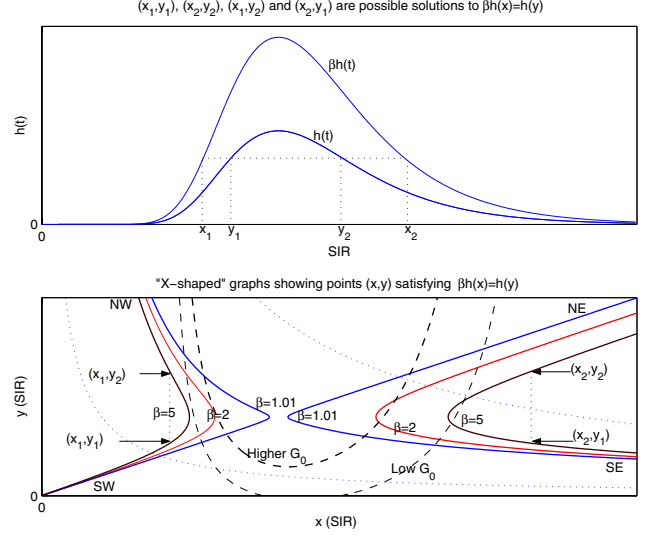

Fig. 2. With x and y respectively the SIR of the favorite and sub-favorite terminals, FONOC requires that $\beta h(x) = h(y)$, with $h(t) = f'(t)\left(1+t/G_0\right)^2$. Any of the min pairs (x_1, y_1), (x_2, y_2), (x_1, y_2), or (x_2, y_1) (top) satisfies this equation, but may not be feasible. We plot all such points, which reveals an "X-shaped" graph for each β. NE, NW, SW and SE are directional labels used to identify the "legs" of the X . On the same axes, we plot the U-shaped graph arising from the constraint equation (21). The 4 intersection points between the U-shaped and X-shaped graphs for the given G_0 and β lead to feasible solutions to FONOC. The NE such point is the best candidate for a maximizer. But if G_0 is "too large", the "U" may lie above the "X" and no intersections would exist. The hyperbolic curves (dotted) represent the constraint equation when *all* terminals operate at maximal bit rate. When G_0 is low, the hyperbola may *only* intersect the SW leg of the X-curve, which leads to a minimum.

sub-figure shows the "X-shaped" graph arising when we plot all points (x, y) satisfying this equation. For a fixed β, this graph has four distinct branches. The NE branch corresponds to points like (x_2, y_2) (top sub-figure), which are both to the right of the peak of $h(t)$. Analogously, the NW, SW and SE branches correspond to points like (x_1, y_2), (x_1, y_1) and (x_2, y_1) respectively.

The "U-shaped" curves at the bottom of fig. (2) are obtained by plotting the constraint equation (21) for various values of G_0 and a given β. But recall that eq. (21) is valid only for certain range of x. The points at which the U-curve (corresponding to the given values of G_0 and β) intersects the corresponding X-curve satisfy both equations (18) and (21), and hence each lead directly to a feasible solution to FONOC.

Increasing β has the effect of "pulling apart" the X-graph, and "widening" the U-curve. Thus, if these graphs intersect for a given β they still intersect after a moderate increase in β. On the other hand, the U-curve "moves up" when G_0 increases, while the X-graph is barely sensitive to changes in G_0. Thus, for G_0 large enough, no intersection (solution) exists. Otherwise, there are four intersection points, one in each of the branches of the concerned X-curve.

We are tempted to prefer the intersection point in the NE branch, where both x (the SIR of the favorite terminal) and y (the SIR of the sub-favorite) are as high as possible. But we

must also consider the non-favored terminals, whose SIR is γ_0, by eq. (13). The throughput of each non-favored terminal is obtained as $f(\gamma_0)/G_1 = f(\gamma_0)/(\gamma_0/\alpha_1^*) \propto \alpha_1^*$. α_1^* is obtained from x^* through eq. (20), which gives rise to a "bell shaped" graph (see comments immediately following eq. (20)). Thus, α_1^* (and hence the throughput of the non-favored terminals) is *decreasing* in x^* beyond a certain value of x^*. Therefore, with possibly many non-favored terminals, it is not obvious that, of the four intersecting points, the one in the NE branch of the X curve leads to the greatest weighted throughput; however, this point does seem the best candidate. This issue merits further research.

IV. EXTENSION AND DISCUSSION

We have investigated the optimal power levels and data rates for terminals transmitting to one base station, in a scenario relevant to 3G CDMA. The objective function is the *weighted* sum of each terminal's throughput. Two weights, which admit various interpretations, including levels of importance, "utilities", or monetary prices, are considered. We have utilized a model which can accommodate many physical layer configurations of practical interest. The properties of the physical layer are embodied in the frame success function (FSF), which gives, in terms of received signal-to-interference ratio (SIR), the probability that a data packet is correctly received. Each physical layer has a preferred SIR, γ_0, easily identified in the graph of the FSF.

Much of the preceding development has focused on the "dual-favorite" boundary solution (DFBS) to the first-order necessary optimizing conditions (FONOC), in which an "important" and an "ordinary" terminal operate at maximal data rate, while other terminals operate at certain optimal SIR, γ_0. This development can be extended to the more general multi-favorite case, in which all the "important" terminals, N_2, and several "ordinary" terminals, say $n_1 \le N_1$, operate at maximal data rate.

The key to obtaining the DFBS is to find a solution to a system of two non-linear equations in two unknowns, (18) and (21), in which the 2 variables are the SIR of the favorite and sub-favorite terminals. Through fig. (2), we described the solution to this system as one of the intersection points of the X-shaped graph (from eq. (18)) with the U-shaped graph (from eq. (21)). In the multi-favorite situation, eq. (18), $\beta h(x) = h(y)$, would apply unmodified, with the understanding that x is the common SIR of all important terminals, and y is the common SIR of all the ordinary terminals operating at maximal data rate. The "non-favored" terminals would again operate with an SIR of γ_0. But eq. (21) would need to be modified slightly as

$$\frac{n_1}{1 + \frac{y}{G_0}} + \frac{N_1 - n_1}{\left(1 + \frac{x}{G_0}\right)\sqrt{\frac{\beta f'(x)}{f'(\gamma_0)}}} + \frac{N_2}{1 + \frac{x}{G_0}} = N_1 + N_2 - 1 \quad (22)$$

The graph of eq. (22) has the same U-shape as that of eq. (21), and is appropriate over a certain range of x as discussed following eq. (20). Therefore, the general discussion summarized in the caption of fig. (2) still applies.

In principle, we would like n_1, the number of "ordinary" terminals operating at maximal data rate, to be as large as possible. However, the U-curve "moves up" when n_1 increases. Thus, for sufficiently large n_1, the sought intersection may *not* exist. Moreover, when we make $n_1 = N_1$ so that all terminals operate at maximal data rate, then the U-curve is replaced by a hyperbolic "L-curve", as displayed in fig. (2). For sufficiently low G_0, the L-curve may intersect only the SW leg of the X-curve, in which both x and y are "low", and this would lead to a *minimum*, not a maximum.

More research is needed before we fully comprehend all interesting aspects of this problem. This includes performing various technical tasks, such as formally proving the shapes of some key graphs, and verifying certain second-order optimality conditions. The issues of QoS requirements, fairness, non-negligible out-of-cell interference, and decentralized implementations are of practical importance and deserve future consideration.

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