Power and Data Rate Assignment for Maximal Weighted Throughput in 3G CDMA

Virgilio Rodriguez, David J. Goodman
Polytechnic University
ECE Department
5 Metrotech Center
Brooklyn, NY 11201

***Abstract*— Relevant to the uplink of a VSG-CDMA system, a technique part of 3G standards, this work seeks power and data rate allocations for each of N terminals, so that the network *weighted* throughput is maximized. The weights admit various interpretations, including levels of importance, "utility", and price. We have learned that at least one terminal should operate at the highest available data rate. Our analysis leads to allocations in which terminals *not* operating at the highest data rate operate at the same signal-to-interference ratio (SIR). This value is determined by the physical layer through the function that gives, in terms of the received SIR, the probability that a data packet is received correctly. Other factors held constant, *lowering* the highest available data rate *increases* the number of terminals which should operate at maximum data rate. This analysis conforms to classical optimization theory. Our model should accommodate a wide variety of physical layer configurations.**

I. Introduction

Modern wireless networks will accommodate simultaneous transceivers operating at very different bit rates. Several technologies have been proposed to accommodate multi-rate traffic in such networks. Ottosson and Svensson [3] discuss several multi-rate schemes based on Direct Sequence Code-Division Multiple Access (DS-CDMA). One such scheme is variable spreading gain (VSG) CDMA, as described, for example, by I and Sabnani[1]. In a VSG-CDMA system, each terminal's spreading gain is determined as the ratio of the common chip rate to the terminal's bit rate.

The model discussed in this paper is relevant to an interference-limited single-cell VSG-CDMA system in which each data terminal can operate within a range of bit rates, which is assumed continuous for tractability. We seek an allocation specifying, for each active terminal, a choice of data rate and power level which will maximize the network weighted throughput. The weights admit various interpretations, including levels of importance or priority, "utilities", or monetary prices (contribution to the network's revenues). The traffic is assumed to be delay-tolerant ("best-effort").

Similar situations have been considered by the literature. Our formulation has much in common with that of Ulukus and Greenstein [9]. Major differences between ours and their work include (a) our consideration of weights (b) our adoption of a "generalized" frame-success function (discussed below), and (c) the simplifying linearization involved in their solution procedure. A recent work by Lee, et al. [2] is also of interest. They also seek data rates and power allocation, and consider a "sigmoidal-like" frame-success function. But they focus on the downlink, do not consider weights, and provide a sub-optimal algorithmic solution based on pricing. Our work has also many similarities with that of Sung and Wong [8]. They maximize a fairly general "capacity function". But they do not consider weights, and assume that the terminal's data rates are fixed exogenous parameters, as opposed to variables to be chosen optimally. Other related works seek decentralized solutions.

Of special notice is our characterization of the frame-success function (FSF), which gives the probability that a data packet is received successfully in terms of the terminal's received signal-to-interference ratio (SIR). This function, which is at the core of the analysis, is determined by physical attributes of the system, including the modulation technique, the forward error detection scheme, the nature of the channel, and properties of the receiver, including its demodulator, decoder, and antenna diversity, if any. We do *not* impose any particular algebraic functional form ("equation") as FSF. Rather, we assume that all that is known about this function is that its graph is a smooth S-shaped curve, as displayed in fig. (1)(see [4] for further discussion of this approach). Our development exploits properties derived from this shape. Hence, our analysis should apply to many physical layer situations of practical interest, as long as they give rise to an FSF with an S-shaped graph.

Below, we first build a relatively simple optimization model relevant to uplink data transmission in one VSG-CDMA cell. Afterward, we provide an outline of the general solution procedure. Then, we discuss the two-terminal special case, as it provides insights useful for the general analysis. Subsequently, we apply the general solution procedure to the N-terminal scenario, and provide some general results. Finally, we discuss our results, and comment on additional research which is needed before we fully comprehend all interesting aspects of this problem.

II. General Formulation

A. Problem Statement

We seek to solve:

$$\text{Maximize} \sum_{i=1}^{N} \beta_i T_i(G_i, \alpha_i) \quad (1)$$

subject to

$$\sum_{i=1}^{N} \frac{\alpha_i}{1+\alpha_i} = 1 \quad (2)$$

$$G_i \geq G_0 \quad i \in \{1, \cdots, N\} \quad (3)$$

In this simple model,

1) N is the number of terminals.
2) The throughput of terminal i is defined as $R_C T_i(G_i, \alpha_i)$, with
$$T_i(G_i, \alpha_i) := \frac{f(G_i \alpha_i)}{G_i} \quad (4)$$
3) $G_i = R_C/R_i$, $i \in \{1, \ldots, N\}$ is the spreading gain of terminal i; i.e., the ratio of the channel's chip rate , R_C to the terminal's data transmission rate R_i (bits per second). $G_0 \geq 1$ is the lowest available spreading gain (determined by the highest available data rate).
4) α_i is the carrier-to-interference ratio (CIR) of the signal from terminal i received at the base station. α_i is defined as,
$$\alpha_i := \frac{P_i h_i}{\sum_{\substack{j=1\\ j\neq i}}^{N} P_j h_j + \sigma^2} = \frac{Q_i}{\sum_{\substack{j=1\\ j\neq i}}^{N} Q_j + \sigma^2} \quad (5)$$
with P_i the transmission power of terminal i, h_i its "gain" (path loss) coefficient, $h_i P_i := Q_i$ its received power, and σ^2 a representative of the average noise power and, possibly, out-of-cell interference. It can be shown that, with $\sigma^2 = 0$, the CIR's must be such that $\sum \alpha_i/(1+\alpha_i) = 1$ (constraint (2)) to ensure feasibility [5].
5) The product $G_i \alpha_i$, denoted as γ_i , is terminal i's signal to interference (SIR) ratio.
6) $\beta_i \geq 1$ is a weight, which admits various practical interpretations. Without loss of generality, we set $1 = \beta_1 \leq \cdots \leq \beta_N$.
7) We assume that there is a real-valued frame-success function (FSF) which gives the probability of the correct reception of a data packet in terms of the received SIR. We assume that this function is such that $f(x) := f_S(x) - f_S(0)$ has the general properties of the generalized "S-curve" introduced in [6] (see fig. (1)), and that it has a continuous second derivative. The difference between f_S and f is generally negligible. Nevertheless, there are good technical reasons for f to be preferred over f_s [4]. It is stressed that no actual function is used, except to provide numerical examples. Our analysis should apply to a wide variety of physical layer configurations, as long as they give rise to an FSF with an S-shaped graph.

It is sometimes useful to observe that constraint (2) can be expressed as

$$\sum_{i=1}^{N} \frac{1}{1+\alpha_i} = N - 1 \quad (6)$$

B. General solution procedure

The general procedure is as follows:

- Create an "augmented" objective function, combining the original objective function with Lagrange multipliers and the constraint equations
- Set up the first-order necessary optimizing conditions (FONOC). This involves setting the partial derivative of the *augmented* objective function with respect to each variable equal to zero. Moreover, inequalities of the form $G_0 - G_i \leq 0$ contribute equations of the form $\mu_i(G_0 - G_i) = 0$, where μ_i is a Lagrange multiplier.
- Solve FONOC. Evidently, each equation of the form $\mu_i(G_0 - G_i) = 0$ requires that if $G_i > G_0$, then μ_i must equal zero; and that if $\mu_i \neq 0$, G_i must equal G_0. Both possibilities must be considered separately while finding various solutions to FONOC.
- A solution to FONOC provides a candidate for a maximizer. The second-order sufficient conditions *may* confirm the candidate as a maximizer. This maximizer may *not* be global.

III. Special Case: N=2

For pedagogical reasons, we first discuss a two-terminal situation. This case is analyzed in greater detail in [7]. Here, we discuss the essential ideas and results.

We seek to solve:

$$\text{Maximize } \frac{f(G_1\alpha_1)}{G_1} + \frac{\beta f(G_2\alpha_2)}{G_2} \quad (7)$$

$$\text{subject to } \alpha_1\alpha_2 = 1 \text{ ; } G_1 \geq G_0 \text{ ; } G_2 \geq G_0$$

It can be easily verified that for N=2, the constraint (2) reduces to $\alpha_1\alpha_2 = 1$. This also follows from the fact that, with negligible noise, $\alpha_1 := Q_1/Q_2 := 1/\alpha_2$.

A. Augmented objective function

Our "augmented" objective function is $\phi(G_1, G_2, \alpha_1, \alpha_2) =$

$$\frac{f(G_1\alpha_1)}{G_1} + \frac{\beta f(G_2\alpha_2)}{G_2} + \lambda(\alpha_1\alpha_2 - 1) + \sum_{i=1}^{2} \mu_i(G_0 - G_i) \quad (8)$$

B. First-Order Necessary Optimizing Conditions (FONOC)

The FONOC can be expressed in vector form, with $\gamma_i = G_i\alpha_i$, as:

$$\begin{bmatrix} \left(\gamma_1 f'(\gamma_1) - f(\gamma_1)\right)/G_1^2 - \mu_1 \\ \beta\left(\gamma_2 f'(\gamma_2) - f(\gamma_2)\right)/G_2^2 - \mu_2 \\ f'(\gamma_1) + \lambda\alpha_2 \\ \beta f'(\gamma_2) + \lambda\alpha_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (9)$$

$$\text{with} \begin{cases} \alpha_1 \alpha_2 = 1 \\ \mu_1 (G_0 - G_1) = 0 \\ \mu_2 (G_0 - G_2) = 0 \end{cases} \tag{10}$$

C. Finding solutions to FONOC

1) Interior ('balanced') solution : First we seek a solution to FONOC which lies in the interior of the feasible region. That is, we presume that a solution exists in which both G_1 and G_2 are greater than G_0, which require $\mu_1 = \mu_2 = 0$ (see equations (10)). Then, we proceed to check whether such a solution actually exists.

Working with the top 2 rows of the matrix equation (9), we obtain $\gamma_i f'(\gamma_i) = f(\gamma_i)$, an equation of the general form:

$$x f'(x) = f(x) \tag{11}$$

Rodriguez[6] shows that for the class of generalized sigmoidal functions, such as f, there is a unique positive value γ_0 which satisfies equation (11). This value can be graphically identified in figure (1) as the abscissa of the point where the graph of f is tangent to a ray emanating from the origin; that is, tangent to the straight line $y = f'(\gamma_0)x$.

Therefore, if any values of the variables of interest satisfy, under the stated hypotheses, equations (9) and (10) , they must be such that:

$$G_1^* \alpha_1^* = G_2^* \alpha_2^* = \gamma_0 \tag{12}$$

By working with the bottom half of the matrix equation (9), we establish that:

$$-\lambda = \frac{f'(G_1^* \alpha_1^*)}{\alpha_2^*} = \frac{\beta f'(G_2^* \alpha_2^*)}{\alpha_1^*} \tag{13}$$

Now, substituting equation (12) into equation (13), we obtain $\alpha_1^*/\alpha_2^* = \beta$, which leads to a complete "interior" solution to FONOC:

$$\begin{aligned} \alpha_1^* = \frac{1}{\alpha_2^*} &= \sqrt{\beta} \\ G_1^* \alpha_1^* = G_2^* \alpha_2^* &= \gamma_0 \end{aligned} \tag{14}$$

Notice that, in order for these values to be feasible, $G_i^* \geq G_0$; i.e., $G_0 \sqrt{\beta} \leq \gamma_0$.

Replacing these values into the objective function yields

$$T_B = \frac{f(\gamma_0)}{G_1^*} + \frac{\beta f(\gamma_0)}{G_2^*} = \frac{f(\gamma_0)\sqrt{\beta}}{\gamma_0} + \frac{\beta f(\gamma_0)}{\gamma_0 \sqrt{\beta}} \tag{15}$$

We have found a closed form solution. If the function f is known, γ_0 can be easily obtained graphically (see figure (1)) or equation(11) can be solved numerically. For instance, for the FSF corresponding, under suitable assumptions, to non-coherent FSK with packet size M=80, which is,

$$f(x) = \left[1 - \frac{1}{2} \exp\left(-\frac{x}{2}\right)\right]^{80} \tag{16}$$

$\gamma_0 = 10.75$, $f(\gamma_0) = 0.83$.

This allocation has an interesting property: it is 'balanced' in the sense that both users experience the same weighted throughput: $f(\gamma_0)\sqrt{\beta}/\gamma_0$. The "fairness" of this operating point may be a desirable feature in certain situations.

Second-order sufficient conditions: The previously found allocation does satisfy FONOC . But it can be shown through the second-order sufficient conditions that it is neither a minimizer nor a maximizer. It is a saddle point [7].

2) An Asymmetric-Rates Boundary Solution (ARBS): In the preceding section, we identified an "interior" solution to FONOC. But this allocation is a non-maximizer, which suggests that a maximizer be sought over the "boundary" of the feasible region; i.e., when $G_i = G_0$ for one or both i. Below, we seek an ARBS solution, in which the "favorite" terminal is the only one transmitting at the highest allowable data rate. That is, we set $G_2 = G_0$, which allows $\mu_2 \neq 0$, and $\mu_1 = 0$, which allows $G_1 \geq G_0$.

Working with the first row of equation (9), and keeping in mind that we have set $\mu_1 = 0$, we obtain $G_1 \alpha_1 = \gamma_0$, with γ_0 as defined by equation (11). Working with the bottom half of equation (9), and using the preceding result, we establish that:

$$\frac{G_0^2 f'(\gamma_0)}{G_0 \alpha_2} = \beta f'(G_0 \alpha_2) G_0 \alpha_2 \quad \Rightarrow \quad \frac{x^2 f'(x)}{f'(\gamma_0)} = \frac{G_0^2}{\beta} \tag{17}$$

with $x := G_0 \alpha_2$. Hence, α_2^* is obtained by solving this equation.

For the class of functions being considered, $x^2 f'(x)$ is a "bell-shaped" function, as shown by figure (1).

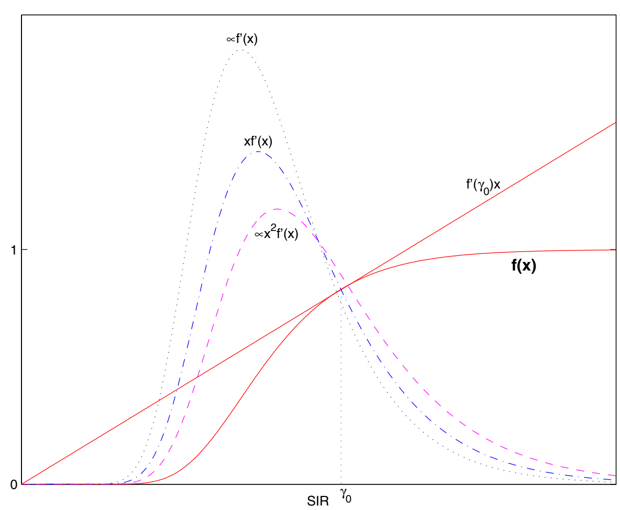

Fig. 1. A particular $f(x)$, $xf'(x)$, and *scaled* versions of $f'(x)$, and $x^2 f'(x)$. γ_0 satisfies $xf'(x) = f(x)$

This implies that, if G_0^2/β surpasses the "peak" of the function on the left hand side of equation (17), then, this equation has *no* solutions. Therefore, if we denote as τ^2 the maximal value (peak) of the function $x^2 f'(x)/f'(\gamma_0)$, (see figure (1)) a condition for the ARBS to exist is that $G_0^2/\beta \leq \tau^2$ or that $G_0 \leq \tau\sqrt{\beta}$. If G_0^2/β is sufficiently small, two values of x will satisfy equation(17). The larger value, to the right of the peak, is chosen as a prospective maximizer, and is denoted as γ_{00}.

In terms of γ_{00} we can identify a complete solution to FONOC. By definition, $\gamma_{00} = G_0\alpha_2$, which implies that $\alpha_2^* = \gamma_{00}/G_0$ satisfies FONOC, and obviously so does $\alpha_1^* = 1/\alpha_2^* = G_0/\gamma_{00}$. And since FONOC requires that $G_1^*\alpha_1^* = \gamma_0$, then G_1^* can be obtained as $\gamma_0/\alpha_1^* = \gamma_0\gamma_{00}/G_0$.

But feasibility requires that $G_1^* \geq G_0$, which imposes that $G_0^2 < \gamma_0\gamma_{00}$. And we already had the condition that $G_0^2 \leq \beta\tau^2$, so G_0^2 cannot exceed $\min\{\gamma_0\gamma_{00}, \beta\tau^2\}$ in order for the ARBS to be feasible.

For example, for the frame success function introduced previously as equation (16), $\gamma_0 = 10.75$, and $f(\gamma_0) = 0.83$. When $G_0 = 2$ and $\beta = 2$, both $x = 22.1$ and $x = 3.97$ satisfy equation (17) . Hence, γ_{00} = 22.1. This gives $T_{ARBS} = 1.01$. By comparison, the 'balanced' solution only yields $T_B = 0.15\sqrt{2} = 0.21$, which is much less.

Second-order sufficient conditions: It can be verified that the allocation we just found in terms of γ_{00}, if feasible, **is a maximizer**.

3) "Greedy" allocation: In the preceding section, we considered the ARBS, in which only the "favorite" terminal operates at the lowest available spreading gain (fastest data rate). It was observed that the ARBS fails to exist if G_0^2/β is "too large", and would be infeasible if $G_0 > \sqrt{\gamma_0\gamma_{00}}$. In this section we seek a "greedy" solution to FONOC, in which both terminals operate at the highest available data rate.

Working with the last two rows of equation (9) we establish that:

$$-\lambda = \frac{f'(\gamma_1)}{\gamma_2} = \frac{\beta f'(\gamma_2)}{\gamma_1} \Rightarrow \gamma_1 f'(\gamma_1) = \beta\gamma_2 f'(\gamma_2) \quad (18)$$

with the constraint $\gamma_1\gamma_2 = G_0^2$.

The symmetric case ($\beta = 1$). To gain insight into the general case, we consider the special case in which β=1. In this case, it is evident that $\gamma_1 = \gamma_2 = G_0$ ($\alpha_1 = \alpha_2 = 1$) (equal received powers) satisfies equation (18), and hence FONOC .

Second-order sufficient conditions: For the class of functions we are considering, the graph of the function $xf'(x)$ is a "bell-shaped" curve, as shown by figure (1). Let $\hat{x}$ be the unique point $\hat{x}$ where $xf'(x)$ reaches its maximum. In [7] we discuss that, when $\beta = 1$, the allocation $\gamma_1 = \gamma_2 = G_0$ is a maximizer for $G_0 > \hat{x}$, but the same allocation is a minimizer if $G_0 < \hat{x}$. In particular, for the function given as equation (16), $xf'(x)$ reaches its maximum at $\hat{x} = 7.95$. Thus, in this example, with $\beta = 1$, $\gamma_1 = \gamma_2 = G_0$ is a maximizer for $G_0 > 7.95$, but a minimizer for $G_0 < 7.95$.

IV. Throughput Optimization with N terminals

A. Augmented objective function

We discuss now the N-terminal situation. The pertinent "augmented" objective function is

$$\phi(G_1, \ldots, G_N, \alpha_1, \ldots, \alpha_N) = \sum_{i=1}^{N} \beta_i T_i(G_i, \alpha_i) + \lambda \left(\sum_{i=1}^{N} \frac{\alpha_i}{1+\alpha_i} - 1 \right) + \sum_{i=1}^{N} \mu_i(G_0 - G_i) \quad (19)$$

B. General First-Order Necessary Optimizing Conditions (FONOC)

The general FONOC can be expressed in vector form, with $\gamma_i = G_i\alpha_i$, as:

$$\begin{bmatrix} \beta_1 \partial T_1(G_1, \alpha_1)/\partial G_1 - \mu_1 \\ \vdots \\ \beta_N \partial T_N(G_N, \alpha_N)/\partial G_N - \mu_N \\ \beta_1 f'(\gamma_1) + \lambda(1+\alpha_1)^{-2} \\ \vdots \\ \beta_N f'(\gamma_N) + \lambda(1+\alpha_N)^{-2} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (20)$$

$$\text{with} \begin{cases} \sum_{i=1}^{N}(1+\alpha_i)^{-1} = N-1 \\ \mu_1(G_0 - G_1) = 0 \\ \vdots \\ \mu_N(G_0 - G_N) = 0 \end{cases} \quad (21)$$

Notice that

$$\frac{\partial T_i(G_i, \alpha_i)}{\partial G_i} = \frac{\gamma_i f'(\gamma_i) - f(\gamma_i)}{G_i^2} \quad (22)$$

C. Solving FONOC

1) Interior solution : First we seek an interior solution to FONOC. That is, we presume that a solution to FONOC exist in which each G_i is greater than G_0, which requires $\mu_i = 0$, (see equations (21)). Then, we proceed to check whether such a solution exists.

Working with the top half of the vector equation (20), we obtain $\gamma_i f'(\gamma_i) = f(\gamma_i)$. Therefore, from the discussion following equation (11), we conclude that, under the stated hypotheses,

$$G_i^* \alpha_i^* = \gamma_0 \quad (23)$$

By working with the bottom half of the vector equation (20), we establish that:

$$-\lambda = \beta_i f'(G_i^*\alpha_i^*)\,(1+\alpha_i^*)^2 = \beta_j f'(G_j^*\alpha_j^*)\left(1+\alpha_j^*\right)^2 \quad (24)$$

Now, substituting equation (23) into equation (24), we obtain :

$$\frac{1}{1+\alpha_j^*} = \frac{\sqrt{\beta_j/\beta_i}}{1+\alpha_i^*} \quad (25)$$

Equation (25) enables us to express α_j^* ($j > 1$) in terms of α_1. This way, the constraint relation (6) becomes an equation which we can solve for α_1^*:

$$\frac{\sum_{j=1}^{N}\sqrt{\beta_j/\beta_1}}{1+\alpha_1^*} = N-1 \Rightarrow \alpha_1^* = \frac{B}{(N-1)} - 1 \quad (26)$$

with $\beta_1 = 1$, and

$$B := \sum_{j=1}^{N} \sqrt{\beta_j} \tag{27}$$

Once we know the value of α_1^*, equation (25) gives us the value of each α_j^*. And once we know each α_i^*, equation (23) gives us immediately the corresponding G_i^* as γ_0/α_i^*. Therefore, we now have a complete "interior" closed-form solution to the first-order optimizing conditions :

$$\alpha_i^* + 1 = \frac{B}{(N-1)\sqrt{\beta_i}} \tag{28}$$

$$G_i^* = \gamma_0/\alpha_i^* \tag{29}$$

Notice that, in order for these values to be feasible, $G_i^* = \gamma_0/\alpha_i^* \geq G_0$ or $\alpha_i^* \leq \gamma_0/G_0$. Under the construction $1 = \beta_1 \leq \cdots \leq \beta_N$, the largest α_i^* is actually α_1^* (see equation (28)). Thus, this condition requires that $B = \sum_{j=1}^{N} \sqrt{\beta_j} \leq (N-1)\,\gamma_0/G_0$.

Substituting the above values into the objective function yields :

$$T_{\text{int-N}} = \sum_{i=1}^{N} \beta_i T_i^* = \frac{f(\gamma_0)}{\gamma_0} \sum_{i=1}^{N} \beta_i \alpha_i^*$$

with

$$\beta_i \alpha_i^* = \frac{B\sqrt{\beta_i}}{N-1} - \beta_i \tag{30}$$

We stress that this a closed form solution. γ_0 can be easily obtained from the graph of function f (see fig. (1)), or equation (11) can be solved numerically.

It is noteworthy that, if $\beta_i = 1$ for all i (terminals are equally "important"), $B = N$ and equation (30) reduces to $1/(N-1)$. Thus, all terminals enjoy the same throughput.

Second-order sufficient conditions It can be shown that the previously found allocation (equations (28 and29)) is neither a maximizer nor a minimizer, but a saddle point.

2) Single-Favorite Boundary Solution (SFBS): We sought and found an allocation satisfying FONOC, where every terminal's data rate is less than the highest available value (equations (29,28)). Unfortunately, this allocation is neither a maximizer, nor a minimizer. This indicates that the true maximizer is a non-interior solution to FONOC; i.e., a solution in which one or more terminals operate at the lowest available spreading gain. In principle, the number of possible non-interior solutions could be very large, of the order of 2^N. A basic rationale is needed to systematically search for these solutions.

A reasonable starting point is to to seek an allocation satisfying FONOC in which only the spreading gain of the "most important" terminal is set at the lowest available value, G_0 (i.e. this terminal operates at the highest available data rate), with other terminals' spreading gains to be determined by the analysis. Below we seek one such solution to FONOC. Our presumption implies that $G_N = G_0$, and that the associated Lagrange multipliers are such that $\mu_i = 0$ for $1 \leq i < N$.

a) General form of SFBS: Working with the first $N-1$ rows of the vector equation (20), and keeping in mind that we have presumed that $\mu_i = 0$ for $1 \leq i < N$, we obtain

$$G_i \alpha_i = \gamma_0 \text{ for } 1 \leq i < N \tag{31}$$

with γ_0 as defined by equation (11), and shown in figure (1).

By working with the bottom half of the vector equation (20), we establish that:

$$-\lambda = \beta_i f'(G_i^* \alpha_i^*)\,(1+\alpha_i^*)^2 \text{ for } 1 \leq i < N \tag{32}$$

and

$$-\lambda = \frac{\beta_N}{G_0^2} f'(x)\,(G_0 + x)^2 \tag{33}$$

with $x := G_0 \alpha_N^*$.

Combining equations (31and 32) we get

$$\frac{1}{1+\alpha_j^*} = \frac{\sqrt{\beta_j/\beta_i}}{1+\alpha_i^*} \text{ for } 1 \leq i\ ,\ j < N \tag{34}$$

Equation (34) enables us to express α_i^* $(1 < i < N)$ in terms of α_1. This way, the constraint relation (6) becomes an equation with only two unknowns, α_1 and α_N. Thus, we can express α_1^* in terms of α_N^*. With

$$B_{N-1} := \sum_{j=1}^{N-1} \sqrt{\beta_j} \tag{35}$$

substituting equation (34) into (6) ($\sum_i (1+\alpha_i)^{-1} = N-1$) yields

$$\frac{B_{N-1}}{1+\alpha_1^*} + \frac{G_0}{G_0+x} = N-1 \quad \rightarrow \tag{36}$$

$$\frac{G_0+x}{1+\alpha_1^*} = \frac{N-1}{B_{N-1}} x + \frac{N-2}{B_{N-1}} G_0 \quad \rightarrow \tag{37}$$

$$\alpha_1^* + 1 = \frac{B_{N-1}}{N-1-(1+\alpha_N^*)^{-1}} \tag{38}$$

Equations (32, and 33) can be combined as ($\beta_1 = 1$):

$$\beta_N f'(x)\,(G_0+x)^2 = G_0^2 f'(\gamma_0)\,(1+\alpha_1^*)^2 \tag{39}$$

which can be put (using equation (37)) as

$$\left(C_1 \frac{x}{G_0} + D_1\right)^2 \frac{f'(x)}{f'(\gamma_0)} = \frac{1}{\beta_N} \tag{40}$$

with

$$C_1 = \frac{N-1}{B_{N-1}}\ ;\ D_1 = \frac{N-2}{B_{N-1}} \tag{41}$$

Let us assume that a meaningful solution to equation (40) can be found, and denote this solution as γ_{00}. In terms of γ_{00} we can identify a complete allocation satisfying FONOC.

By definition, $\gamma_{00} = G_0 \alpha_N^*$ which implies that $\alpha_N^* = \gamma_{00}/G_0$ satisfies FONOC. From α_N^*, equation (38) gives us immediately α_1^*, and from α_1^* we can obtain each α_i^* $(1 < i < N)$ through equation (34). And since each G_i^* $(1 \leq i < N)$ must satisfy $G_i^* \alpha_i^* = \gamma_0$ (equation (31)), once each α_i^* $(1 < i < N)$ is known, so is the corresponding G_i^*. The complete allocation is given by:

$$
\begin{aligned}
G_N^* &= G_0 &(42)\\
G_0\alpha_N^* = \gamma_N^* &= \gamma_{00} &(43)\\
\text{for } 1 \le i < N & & \\
\alpha_i^* + 1 &= \frac{1}{\sqrt{\beta_i}} \frac{B_{N-1}}{N - 1 - (1+\alpha_N^*)^{-1}} &(44)\\
G_i^*\alpha_i^* = \gamma_i^* &= \gamma_0 &(45)
\end{aligned}
$$

Notice, however, that each G_i^* must satisfy $G_i^* \ge G_0$ or $\alpha_i^* \le \gamma_0/G_0$.

b) Existence of this solution: The preceding allocation depends on a solution to the single-variable algebraic equation (40). Here we examine the conditions under which this algebraic equation has solution(s), and if it does, which one of its solutions should be chosen.

Observe, first, that $C_1x/G_0 + D_1 \le x + 1$. This is so, because the left-hand side of this inequality is largest when G_0 and B_{N-1} are smallest (see equations (41)). Because of technological limitations, $G_0 \ge 1$ (the highest available data rate cannot exceed the channel's "chip rate"). And, $B_{N-1} = \sum_{j=1}^{N-1} \sqrt{\beta_j} \ge N - 1$, since, by construction, $1 = \beta_1 \le \beta_i$ for $\forall i$. Hence, $C_1 \le 1$ and $D_1 \le (N-2)/(N-1) \le 1$. All this implies that $C_1x/G_0 + D_1 \le x + 1$.

It can be shown that, as displayed by figure (1), for the class of functions being considered, the graph of the function $x^2 f'(x)$ is "bell-shaped", and so is the graph of $(x+1)^2 f'(x)/f'(\gamma_0)$. On the basis of the preceding paragraph, it can be further concluded that the function $(C_1x/G_0 + D_1)^2 f'(x)/f'(\gamma_0)$ is also bell-shaped. This implies that, if G_0 is "too large", the "peak" of this function may fall below $1/\beta_N$, unless β_N is also "very large". Thus, equation (40) may have no solution. On the other hand, when G_0 is sufficiently small and/or β_N is sufficiently large, two values of x, on either side of the peak of the concerned function, will satisfy equation (17). The larger value is chosen as the prospective maximizer called γ_{00} in the preceding subsection. Equations (42-45) give a complete solution to FONOC in terms of γ_{00} .

3) Dual-Favorite Boundary Solution: As discussed in section IV-C.2.b, there may *not* be a feasible solution to FONOC in which the favorite terminal is the *only* one operating at the highest available data rate. In this section we explore the existence of a boundary allocation satisfying FONOC, in which *both* the favorite, and second-favorite terminals operate at the highest data rate. That is, we set $G_N = G_{N-1} = G_0$, and $\mu_i = 0$ for $1 \le i \le N-2$, and determine under which conditions, if any, a solution to FONOC satisfying these hypotheses actually exists.

Proceeding as in section IV-C.2, we determine that, for the special case in which both favorite terminals have the same weight, β_N, in order for the desired solution to exist, the SIR of the "non-favorite" terminals should be γ_0, and the SIR of the favorites should be a solution to the algebraic equation:

$$\left(C_2\frac{x}{G_0} + D_2\right)^2 \frac{f'(x)}{f'(\gamma_0)} = \frac{1}{\beta_N} \qquad (46)$$

with $B_{N-2} = \sum_{j=1}^{N-2} \sqrt{\beta_j}$ and

$$C_2 = \frac{N-1}{B_{N-2}} \; ; \; D_2 = \frac{N-3}{B_{N-2}} \qquad (47)$$

But notice that this equation is nearly identical to equation (40). The only difference is that the constants C_2 and D_2 replace C_1 and D_1 (equation (41)). Accordingly, the discussion of section IV-C.2.b also applies to this case. Thus, we know that a meaningful solution to equation (46) exists under conditions analogous to those given in section IV-C.2.b for the existence of a solution to equation (40). Notice also that $B_{N-2} < B_{N-1}$ which tends to make the left-hand side of equation (46) larger than the left-hand side of equation (40) (in particular, $C_2 > C_1$). This means that, for fixed G_0 and β_N, equation (46) may have solutions even if (40) does *not*.

Once a meaningful solution to equation (46) has been found, following a development analogous to that leading to equations (42, 43, 44, and 45), we can obtain a complete "dual favorite" solution to FONOC.

V. Discussion

We have sought the optimum power levels and data rates for terminals transmitting to one base station, in a scenario relevant to 3G CDMA. The objective function is the *weighted* sum of each terminals' throughput. These weights admit various interpretations, including levels of importance, "utilities", or monetary prices. We have utilized a model which can accommodate many physical layer configurations of practical interest. The properties of the physical layer are embodied in the frame success function (FSF), which gives, in terms of received signal-to-interference ratio (SIR) the probability that a data packet is correctly received. Each physical layer has a preferred SIR, γ_0, easily identified in the graph of the FSF.

The 2-terminal special case makes us focus on 3 assignments: (i) a "balanced" allocation, in which both terminals operate at γ_0, and achieve equal weighted throughput; (ii) an "unfair" assignment in which the favorite terminal operates at maximum data rate, with the other terminal achieving the optimal SIR, γ_0; and (iii) a "greedy" assignment in which both terminals operate at maximal bit rate. The balanced assignment is always suboptimal, implying that "fairness" comes at the expense of performance, in this context. The favorite terminal should always operate at maximal data rate. Only when the ratio $G_0/\sqrt{\beta}$ is larger than certain threshold determined by the physical layer through the FSF should both terminals operate at maximal data rate (G_0 is the smallest available spreading gain and β is the weight of the favorite terminal).

The "greedy" allocation is particularly treacherous, which can be shown clearly when both terminals are equally weighted. In this case, an equal-received-power assignment satisfies the first-order necessary optimizing conditions (FONOC). But this assignment can lead to either a maximum or a minimum, depending upon whether G_0 exceeds a specific value determined by the physical layer. It is significant that the greedy and the unfair allocations are complementary in this sense: A low G_0 may turn the greedy allocation into a minimizer, but the unfair allocation, which is a maximizer, needs a low G_0 in order to be feasible.

With N terminals, the situation is more opaque, and necessitates additional research. Nevertheless, our results provide useful guidance. We have identified an "interior" solution to FONOC in which all terminals achieve the optimal SIR, γ_0, referred to above, and operate with data rates below the highest available. This solution is "fair" at least when terminals are equally weighted; but it is suboptimal (a saddle point). Thus, one or more terminals should operate at the highest available data rate. But, for a large N, many such allocations are possible.

A reasonable starting point for examining these solutions is to seek an allocation satisfying FONOC in which *only* the favorite terminal operates at the highest available data rate. Our analysis shows that in order for this "single favorite" solution to exist, the SIR of the "non-favorite" terminals should be the previously mentioned γ_0, and the SIR of the favorite terminal should be a solution to the equation $(C_1 x/G_0 + D_1)^2 f'(x)/f'(\gamma_0) = 1/\beta_N$. In this equation, f is the FSF, $\beta_N \geq 1$ is the weight of the favorite terminal, and C_1 and D_1 are constants which are largest when the weights of the non-favorite terminals are smallest. But the function $(C_1 x/G_0 + D_1)^2 f'(x)/f'(\gamma_0)$ is "bell-shaped". Thus, if G_0 is "too large" the "peak" of this function may fall below $1/\beta_N$, unless β_N is also "very large". All this makes intuitive sense. When G_0 is "very large", the highest available data rate is relatively low, so keeping only one terminal operating at the highest data rate is *not* appealing, unless that terminal has "a lot more weight" than the others. However, when G_0 is "very small", the highest available data rate is very high, and an allocation in which *only one* terminal operates at this very high data rate is more appealing .

When a "single favorite" solution to FONOC is not possible, a natural step is to seek a "dual favorite" solution. Doing so led us to a situation very similar to what we just described. The non-favorite terminals should operate with an SIR of γ_0, and the SIR of the 2 favorites should be a solution to an equation analogous to that discussed in the previous paragraph. Even if the previous equation has no solution, this equation *may* have solutions. But if G_0 is sufficiently large, the dual-favorite solution to FONOC may also fail to exist. In this case, we would seek a "triple favorite" solution. And so on.

Other factors held constant, *lowering* the highest available data rate *increases* the number of terminals which should operate at maximum data rate. It is noteworthy that terminals *not* operating at maximal data rate should still achieve the preferred SIR of γ_0, which is a respectable value. For example, for a simple, but plausible FSF (equation (16)), $f(\gamma_0) = 0.83$. Thus, even "non-favorite" terminals enjoy reasonable error performance.

More research is needed before we fully comprehend all interesting aspects of this problem. This includes various technical tasks, such as verifying certain second-order optimality conditions, which are essential to ascertain that a solution to FONOC is actually a maximizer. This involves showing that certain matrices are positive definite. But with an arbitrary FSF, and symbolic parameters (N, G_0, β_i , etc.), these matrices are symbolic, which complicates this matter. Consideration of interesting special cases could enhance our intuition, as would additional numerical exercises. The issues of QoS requirements, fairness, non-negligible out-of-cell interference, and decentralized implementations are of practical importance and deserve future consideration.

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REFERENCES

[1] I, Chih-Lin and K.K. Sabnani, "Variable spreading gain CDMA with adaptive control for true packet switching wireless network", IEEE ICC, pp: 725 -730 vol.2, 1995

[2] Lee, J.W., R.R. Mazumdar and N.B. Shroff, "Joint Power and Data Rate Allocation for the Downlink in Multi-class CDMA Wireless Networks", Proc. of the 40th Allerton Confer. on Comm., Control and Comp., Oct. 2002

[3] Ottosson, T. , and A. Svensson, "Multi-rate schemes in DS/CDMA systems", IEEE Vehicular Tech. Conf. , pp: 1006 -1010, vol.2, 1995

[4] Rodriguez, V., "Robust Modeling and Analysis for Wireless Data Resource Management" To appear, IEEE WCNC, 2003

[5] Rodriguez, V., "From Power Levels to Power Ratios and Back: A Change of Domain and its Modeling Implications", WICAT Tech. Rep. 02-011, Polytechnic Univ., 2002 `http://wicat.poly.edu/reports`

[6] Rodriguez, V., "Maximizing the Ratio of a Generalized Sigmoid to its Argument", WICAT Tech. Rep. 02-010, Polytechnic University, 2002 `http://wicat.poly.edu/reports`

[7] Rodriguez, V., and D.J. Goodman "Prioritized Throughput Maximization via Rate and Power Control for 3G CDMA: The 2 Terminal Scenario", Proc. of the 40th Allerton Confer. on Comm., Control and Comp., Oct. 2002

[8] C. W. Sung and W. S. Wong, "Power Control and Rate Management for Wireless Multimedia CDMA Systems", IEEE Trans. Commun., vol. 49, no. 7, pp. 1215-26, July 2002

[9] Ulukus, S. and L.J. Greenstein "Throughput maximization in CDMA uplinks using adaptive spreading and power control" IEEE ISSSTA 2000, pp: 565 -569 vol.2