MAXIMIZING THE THROUGHPUT OF CDMA DATA COMMUNICATIONS

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***Abstract*—We analyze aggregate throughput as a function of the transmitter power levels and the number of terminals sending data to a CDMA base station. We find that when noise and out-of-cell interference are negligible, received power balancing maximizes the aggregate throughout of the base station, provided the population of active terminals does not exceed an optimum size. The optimum number of active terminals depends on the CDMA processing gain and the details of the physical layer and data link layer. These details are summarized in a univariate frame success function. When noise is present, power balancing is suboptimal mathematically but attractive for practical implementation.**

Keywords-power control; radio resources management; power balancing

I. INTRODUCTION

We analyze the throughput of a CDMA base station receiving data from N transmitters, all operating at the same constant bit rate. We consider two resource management issues: transmitter power control and the number of terminals that should be admitted to the system in order to maximize base station throughput.

Early work on uplink CDMA power control focused on telephone communications and determined that to maximize the number of voice communications, all signals should arrive at a base station with equal power [1]. Initial studies of power control for data communications focused on maximizing the utility of each terminal, with utility measured as bits delivered per Joule of radiated energy [2,3]. Our recent work [4-7] adjusts the power and rate of each terminal to maximize $\Sigma_i \beta_i T_i$, the aggregate weighted throughput of a base station. T_i b/s denotes the throughput of terminal i and the weight β_i admits various interpretations, such as priority, utility per bit, or a monetary price paid by the terminal. This work assumes that the number of active terminals is fixed, their data rates are continuous variables to be optimized, and that the system is interference limited (noise is negligible). In the present paper, we set all the weights equal to unity, and take the data rates to be identical and fixed, but we view the number of active terminals as a key variable to be optimized. We also consider random noise to be non-negligible, which is essential when out-of-cell interference is significant, and included in the noise term.

Other authors have considered situations relevant to ours. For example, Ulukus and Greenstein [8] adjust data rates and transmitter power levels in order to maximize network throughput. Lee, Mazumdar, and Shroff [9] adapt data rates and power allocation for the downlink, and provide a sub-optimal algorithmic solution based on pricing. Sung and Wong [10] assume that the terminals' data rates are different but fixed, and maximize a capacity function.

The research reported in this paper finds that there is an optimum number of active terminals. When noise and out-of-cell interference* are negligible, the transmitter power levels should be controlled to achieve *power balancing*. With power balancing all signals arrive at the base station with equal power. On the other hand, when noise and interference from other cells are not negligible, the mathematical power optimization problem is more complicated. The optimum set of transmitter powers depends on the maximum achievable signal-to-noise ratios of the N terminals. The optimum received power levels are unequal. Nevertheless power balancing remains attractive for practical implementations.

We also find that in order to maximize base station throughput with any power control algorithm, the number of active transmitters, N, should be limited to $N \leq N^*$, where N^*, is a property of the *frame success function* $f(\gamma)$, the probability that a terminal's data packet is received successfully as a function of γ, the received signal-to-interference-plus-noise ratio (SINR). The specific form of $f(\gamma)$ depends on the details of the CDMA transmission system including the bandwidth, packet size, modem configuration, channel coding, antennas, and radio propagation details. Our analysis applies to a wide class of practical frame success functions, each characterized by a smooth S-shaped curve [4]. ,

* To be concise we refer to the combination of noise and inter-cell interference simply as "noise". The analysis does not distinguish the two impairments. It considers only their combined power.

The next section presents the CDMA transmission system and a statement of the throughput optimization problem. The analysis in Section III assumes that the performance of each link is limited by interference from other transmissions in the same cell (no additive noise and no interference from other cells). Section IV analyzes the effects of noise and out-of-cell interference.

II. THE OPTIMIZATION PROBLEM

A data source generates packets of length L bits at each terminal of a CDMA system. A forward error correction encoder, if present, and a cyclic redundancy check (CRC) encoder together expand the packet size to M bits. The data rate of the coded packets is R_s b/s. The digital modulator spreads the signal to produce R_c chips/s. The CDMA processing gain is $G=W/R_s$, where W Hz, the system bandwidth, is proportional to R_c. Terminal i also contains a radio modulator and a transmitter radiating P_i watts. The path gain from transmitter i to the base station is h_i and the signal from terminal i arrives at the base station at a received power level of $Q_i=P_ih_i$ watts. The base station also receives noise and out-of-cell interference with a total power of σ^2 watts. The base station has N receivers, each containing of a demodulator, a correlator for despreading the received signal, and a cyclic redundancy check decoder. Each receiver also contains a channel decoder if the transmitter includes forward error correction.

In our analysis, the details of the transmission system are embodied in a mathematical function $f(\gamma)$, the probability that a packet arrives without errors at the CRC decoder. The dependent variable γ, is the received SINR. For terminal i,

$$\gamma_i = G\frac{P_ih_i}{\sum_{\substack{j=1\\ j\neq i}}^{N} P_jh_j+\sigma^2} = G\frac{Q_i}{\sum_{\substack{j=1\\ j\neq i}}^{N} Q_j+\sigma^2} \tag{1}$$

Acknowledgment messages from the receiver inform the transmitter of errors detected at the CRC decoder that have not been corrected by the channel decoder. The transmitter employs selective-repeat retransmission of packets received in error.

In parts of our analysis we assume that intra-cell interference dominates the total distortion and study system performance when $\sigma^2=0$. When $\sigma^2>0$, we define the signal-to-noise ratio of receiver i as $s_i=Q_i/\sigma^2$ and rewrite Equation (1) as

$$\gamma_i = G\frac{s_i}{\sum_{\substack{j=1\\ j\neq i}}^{N} s_j+1}. \tag{2}$$

In cases of practical interest $f(\gamma)$ is a continuous, increasing S-shaped function of γ, with $f(0)=2^{-M}\approx 0$ and $f(\infty)=1$ [4].

If the probability of undetected errors at the CRC decoder is negligible, the throughput of signal i, defined as the number of information bits per second received without error, is:

$$T_i = \frac{L}{M}R_s f(\gamma_i) \quad \text{b/s}, \tag{3}$$

The aggregate throughput, T_{total}, is the sum of the N individual throughput measures in Equation (3). Assuming that L, M, and R_s are system constants, we analyze the normalized throughput U defined as

$$U = \frac{M}{LR_s}T_{total} = \sum_{i=1}^{N} f(\gamma_i)\,. \tag{4}$$

U is dimensionless and bounded by $0\leq U\leq N$.

The aim of our optimization study is to find the transmitter power levels, P_i, that maximize U given N, the number of terminals transmitting simultaneously. We then examine the maximum throughput as a function of N in order to find the number of simultaneous transmitters that results in the highest normalized throughput. To find the optimum transmitter power levels, it is convenient mathematically to maximize Equation (4) with respect to the received powers $Q_i, Q_2,\ldots,Q_N$. To do so, we differentiate Equation (4) with respect to each of the received power levels Q_i. We then examine the N derivatives under the power balancing condition $Q_i=Q$ for $i=1,2,\ldots,N$. Under this condition, all of the derivatives are equal. They have the following properties.

$$\left.\frac{\partial U}{\partial Q_i}\right|_{Q_i=Q} = 0;\ \ \sigma^2=0; \qquad \left.\frac{\partial U}{\partial Q_i}\right|_{Q_i=Q} > 0;\ \ \sigma^2>0 \tag{5}$$

These formulas indicate that when performance is limited by intra-cell interference ($\sigma^2=0$), it is possible that maximum throughput occurs when all signals arrive at the base station at the same power level. The optimization problem is more complex when $\sigma^2>0$.

III. MAXIMUM THROUGHPUT, NO ADDITIVE NOISE

Before analyzing performance for arbitrary values of N, we examine the two-terminal case ($N=2$) to gain insight into the effects of power levels on base station throughput.

A. Two terminals

The simplest non-trivial maximization of Equation (4) occurs when $N=2$ and $\sigma^2=0$ [5]. In this case the normalized throughput U is a function of just one variable, $z=Q_2/Q_1$. Moreover, $\gamma_2=Gz$ and $\gamma_1=G/z$. Adopting the notation $U_2(z)=f(Gz)+f(G/z)$ as the normalized throughput when $N=2$ and $\sigma^2=0$, we find that $dU_2/dz=0$ at $z=1$. This suggests that $U_2(1)=2f(G)$ could be a local maximum or a local minimum, depending on the sign of the second derivative at $z=1$. Examining the second derivative for arbitrary S-shaped $f(\gamma)$, we find that $U_2(1)$ is a local minimum at low values of G and a local maximum at high values. Specifically,

$$\left.\frac{d^2U_2}{dz^2}\right|_{z=1} = 2G[Gf''(G) + f'(G)] \equiv 2G\frac{d}{dx}\left(xf'(x)\right)\Big|_{x=G} \tag{6}$$

Therefore, the sign of the second derivative of U_2 is the same as that of the derivative of $xf'(x)$. for the class of functions $f(x)$, $xf'(x)$ has a bell shape reaching a single maximum at some point, $x=G^*$. Therefore, the derivative of $xf'(x)$ is positive for any x to the left of G^*, and is negative otherwise. This means that $U_2(1)$ is a local maximum when G is large enough to exceed G^*, and is a local minimum otherwise.

When $z=1$ yields a local minimum, we must determine whether there is a global maximum at the boundaries of the feasible region; i.e. at $z=0$ and $z=\infty$. Even when $z=1$ is a local maximum, we must consider the possibility that the *global* maximum lies at the boundary of the feasible region. This suggests that we compare the equal-received-power condition ($z=1$) with the condition that terminal 2 does not transmit at all ($z=0$). The comparison reveals that $z=1$ produces higher throughput when $2f(G)>1$, while $z=0$ produces higher throughput when $2f(G)<1$. Accordingly we define the *critical processing gain*, G_c as the value of G for which $f(G)=0.5$. G_c has the property that single terminal transmission ($z=0$ or $z=\infty$) is better than equal-received-power transmission when $G<G_c$ and conversely, when $G>G_c$. Because G is proportional to bandwidth, we can state that the system requires a bandwidth corresponding to at least $G=G_c$ to support two data terminals.

As a numerical example, we refer to the frame success probability for the non-coherent frequency shift keying modem and the frame size $M=80$ considered in our previous work [2-7]. In this case

$$f(\gamma)=[1-0.5exp(-\gamma/2)]^{80} \tag{7}$$

and $f(G)=0.5$ at a processing gain $G=G_c=8.12$. Likewise, $G^*=7.95$ (when the processing gain exceeds this value, $z=1$ is a local maximizer). This suggests that with a processing gain $G\leq 8$ it would be better to turn off the transmitter in one of the terminals and let the other terminal use the entire base station. With $G\geq 9$, it would be better to have both terminals transmitting to achieve equal received power. **Figure 1** confirms that this is the case. It shows $U_2(z)$, normalized throughput as a function of the received power ratio (with z plotted on a log scale), for six values of processing gain.

At the equal power condition ($z=1$), we observe that $U_2(1)<1=U_2(0)$ for $G=4$, $G=7$ and $G=8$. When $G=9$, $G=16$, and $G=20$, $U_2(1)>1$.

B. Arbitrary number of terminals

The first part of Equation (5) encourages us to explore power balancing to determine whether it provides maximum throughput or minimum throughput. Rather than examine second derivatives, we extend the approach adopted in the previous Section for $N=2$. To do so, we assume that signals f from K transmitters arrive at the base station with equal power and that the other N-K terminals turn off their transmitters.

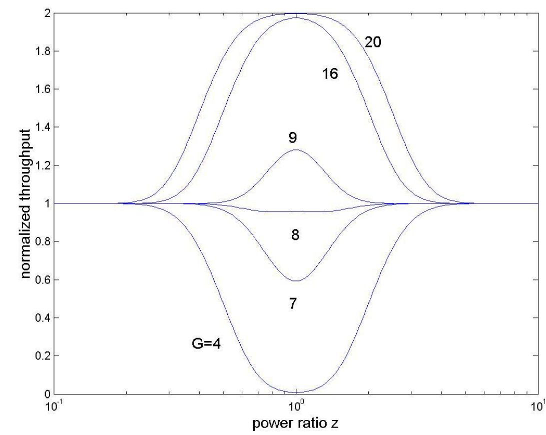

Figure 1: If the processing gain is at least $G=9$, the normalized throughput is maximum when the two transmitters are active. Otherwise, it is better to turn one of them off.

First we observe that when $Q_i=Q$ for $i=1,2,\ldots,K$ and $\sigma^2=0$, Equations (1) and (3) imply

$$U(K) = Kf\left(\frac{G}{K-1}\right). \tag{8}$$

Here, the notation $U(K)$ refers to the power balancing throughput as a function of K. We observe that for values of $G>G_c$ this function has a maximum value for an integer $K=N^*\geq 2$. We infer that the throughput is maximum when N^* signals are received with equal power and the other N-N^* signals are not transmitted at all. Our analysis leads to the observation that maximizing Equation (8) with respect to K is equivalent to maximizing $f(\gamma)/\gamma$. For the class of functions f being considered, $f(x)/x$ has a unique maximum at the point where a line from the origin is tangent to $f(x)$ [4]. We use the notation γ^* for the signal-to-interference ratio that maximizes $f(x)/x$. γ^* is the unique solution to the equation

$$f(\gamma) = \gamma\frac{df(\gamma)}{d\gamma}. \tag{9}$$

For maximum throughput, the system should operate with a value of K that produces $\gamma=G/(K-1)\approx\gamma^*$. Since K has to be an integer, we infer that N^* is the integer just above or just below $1+G/\gamma^*$. In our numerical example with the frame success probability function in Equation (7), $\gamma^*=10.75$. For example, with $G=128$, $1+G/\gamma^*=12.90$. In Figure 2, we have $U(12)=10.65$, $U(13)=10.71$, and $U(14)=10.46$, which implies $N^*=13$.

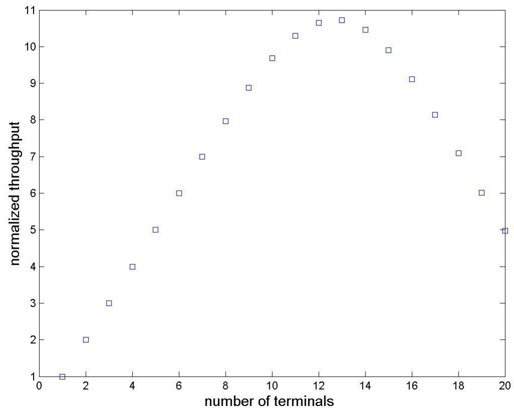

Figure 2: With processing gain $G=128$, the analysis predicts that the optimum number of terminals $N^*\approx 12.90$. The graph confirms this. Throughput is maximum with $N=N^*=13$.

IV. MAXIMUM THROUGHPUT WITH NOISE PRESENT

We now expand the study to take into account the effects of additive noise and interference from other cells. The total power in these impairments is σ^2 watts (and we refer to them together as "noise" to be concise). The noise appears at the receiver as an additional signal that does not contribute to the overall throughput. The system has to use some of its power and bandwidth resources to overcome the effects of the noise.

The effects of noise depend on the power limits of practical terminals. With unlimited power, we would increase all the received powers Q_i indefinitely until the effect of the noise is negligible. To account for the power limits, let $P_{i,max}$ denote power of the strongest possible signal transmitted by terminal i and $Q_{i,max}=P_{i,max}h_i$, the corresponding received signal. The maximum signal-to-noise ratio of terminal i is $s_{i,max}=Q_{i,max}/\sigma^2$. In our analysis, we order the labels of the terminals such that $Q_{1,max}\geq Q_{2,max}\geq\ldots\geq Q_{N,msax}$. In many situations this ordering implies that terminal 1 is closest to the base station and terminal N is most distant.

A. Two terminals

With $\sigma^2=0$ and $N=2$, we derived two principal conclusions: 1) there is a critical processing gain, G_c, that permits two terminals to share the channel with higher total throughput than one terminal alone can achieve, and 2) when $G>G_c$, the throughput is maximum when both signals arrive at the base station with equal power. This section explores the same issues in the presence of additive noise, $\sigma^2>0$.

With $Q_{2,nax}\leq Q_{1,max}$, it is reasonable to assume that when terminal 2 is admitted to the system, it transmits with maximum power to achieve $Q_2=Q_{2,max}$. To explore power balancing, we find that $\partial U/\partial Q_1>0$ at $Q_1=Q_{2,max}$. This implies that higher throughput can be achieved at a value of $Q_1>Q_{2,max}$ than with $Q_1=Q_{2,max}$. Thus, we conclude that with $\sigma^2>0$, power balancing is sub-optimal and the throughput with $Q_1=Q_2$ is a lower bound on the maximum possible throughput.

When $Q_1=Q_2=Q_{2,max}$ the lower bound is

$$U=U_2=2f\left(G\frac{s_{2,\max}}{s_{2,\max}+1}\right). \tag{10}$$

When $Q_2=0,\ U=U_1=f(Gs_1)\leq 1$. It follows that a sufficient condition for admitting two terminals is $U_2\geq 1$. We denote the processing gain necessary to meet this condition as G_{cc}. Equation (10) implies that the minimum G_{cc} that achieves $U_2\geq 1$ satisfies:

$$f\left(G_{cc}\frac{s_{2,\max}}{s_{2,\max}+1}\right)=0.5. \tag{11}$$

Recalling that with $\sigma^2=0$, the critical processing G_c satisfies $f(G_c)=0.5$, we conclude that noise increases the bandwidth sufficient for admitting two terminals by the factor $(s_{2,\max}+1)/s_{2,\max}$.

Another way to assess the effects of noise is to consider the processing gain fixed at $G>G_c$ and examine Equation (10) to find the value of $s_{2,max}$ sufficient for $U_2\geq 1$. This analysis leads to the conclusion that a sufficient condition for admitting two terminals is:

$$s_{2,\max}\geq s_{cc}=\frac{G_c}{G-G_c}. \tag{12}$$

This critical SNR corresponds to a critical distance, d_c meters between transmitter 2 and the base station. When the actual distance, $d>d_c$, system throughput is higher when $Q_2=0$. To determine d_c, recall that $s_{2,max}=P_{2,max}h_2/\sigma^2$, where h_2 is the distance-dependent path gain of terminal 2. Referring to a simple propagation model in which $h_2=const/(d_2)^{\alpha}$, we adopt the policy $Q_2=Q_{2,max}$ when

$$d\leq d_c=\left(const\frac{P_{2,\max}}{\sigma^2}\frac{G-G_c}{G_c}\right)^{1/\alpha}; \tag{13}$$

otherwise $Q_2=0$.

Figure 3 illustrates the tradeoff between processing gain and operating range for the system studied in the other numerical examples in this paper, in which $G_c=8.12$. The graph pertains to $\alpha=3.5$ and $constP_{max}/\sigma^2=10^{10}$. This scaling factor implies that a terminal transmitting with maximum power at a distance 100 meters from the receiver achieves SNR $s_2=1000$. In Figure 3, the operating range increases rapidly as a function of processing gain as G increases from $G=G_c=8.12$ to $G\approx 15$. Then, further increases in processing gain bring a more gradual increase in operating range.

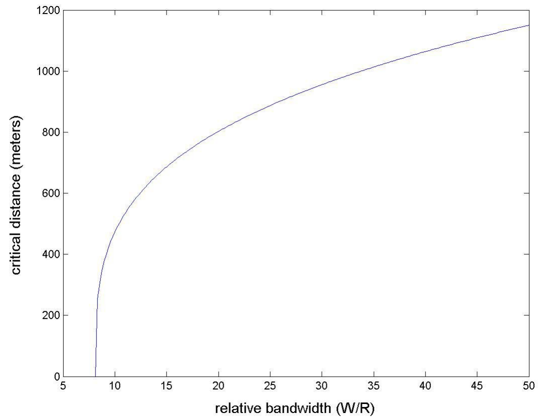

Figure 3: Operating range of the weaker terminal as a function of processing gain, $G=W/R_s$. As the processing gain increases, the terminal can transmit from a longer distance with throughput ≥1.

B. Arbitrary Number of Terminals

Extending the analysis for $N=2$ terminals, we assume that terminal N, with minimum $Q_{i,max}$, transmits at maximum power, $P_N=P_{N,max}$ and achieves a signal–to-noise ratio $s_{N,max}$. If all other terminals adjust their transmitter powers to achieve $Q_i=Q_{N,max}$, the normalized throughput, according to Equations (2) and (4) is

$$U = Nf\left(G\frac{s_{N,\max}}{(N-1)s_{N,\max}+1}\right) \tag{14}$$

Comparing this formula with Equation (8) we observe that to overcome the effects of noise, the system could operate with a bandwidth sufficient to produce a processing gain

$$G' = G\left(1+\frac{1}{(N-1)s_{N,\max}}\right). \tag{15}$$

Therefore, to overcome the effects of noise, the system could expand its bandwidth by the factor $1+1[(N-1)s_{N,max}]$. Alternatively, the system could maintain the optimum throughput per terminal of the noiseless system by reducing the number of active transmitters from N to $N-1/s_{N,max}$. This suggests that the noise appears to the system as $1/s_{N,max}$ interfering transmitters that make no contribution to the aggregate throughput.

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REFERENCES

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[1] R.D. Yates, “A framework for uplink power control in cellular radio systems", IEEE Journal on Selected Areas in Communications, vol. 13, no. 7, pp. 1341--1347, September 1995.

[2] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, “Efficient power control via pricing in wireless data networks”, IEEE Transactions on Communications, vol. 50, no. 2, pp. 291-303, Feb.2002.

[3] D. Goodman and N. Mandayam, “Network assisted power control for wireless data”, *Mobile Networks and Applications*, vol. 6, no. 5, pp. 409-418, Sept, 2001.

[4] Rodriguez, V., "Robust modeling and analysis for wireless data resource management" IEEE WCNC, Vol. 2 , pp. 717 -22 , March 2003

[5] Rodriguez, V., and D.J. Goodman "Prioritized throughput maximization via rate and power control for 3G CDMA: the 2 terminal scenario", Proc. of 40th Allerton Conf. on Comm., Control and Comp., Oct. 2002

[6] Rodriguez, V., and D.J. Goodman, "Power and data rate assignment for maximal weighted throughput in 3G CDMA" IEEE WCNC, vol. 1, pp. 525 -31, March 2003

[7] Rodriguez, V., and D.J. Goodman, "Power and Data Rate Assignment for Maximal Weighted Throughput: A Dual-Class 3G CDMA Scenario" IEEE ICC, vol. 1 , pp. 397 -401, May 2003

[8] Ulukus, S. and L.J. Greenstein "Throughput maximization in CDMA uplinks using adaptive spreading and power control" IEEE ISSSTA, vol. 2, pp: 565 -569, 2000

[9] Lee, J.W., R.R. Mazumdar and N.B. Shroff, "Joint power and data rate allocation for the downlink in multi-class CDMA wireless networks", Proc. of 40th Allerton Conf. on Comm., Control and Comp., Oct. 2002.

[10] C. W. Sung and W. S. Wong, "Power control and rate management for wireless multimedia CDMA systems", IEEE Trans. Commun., vol. 49, no. 7, pp. 1215-26, July 2002