Capacity and power control in spread spectrum macro-diversity radio networks revisited

V. Rodriguez, **RUDOLPH MATHAR**, A. Schmeink

Theoretische Informationstechnik
RWTH Aachen
Aachen, Germany
email: vr@ieee.org, {mathar@ti , schmeink@umic}.rwth-aachen.de

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Outline

Macro-diversity

- Macro-diversity [1]:
  - cellular structure is removed
  - each transmitter is jointly decoded by all receivers (RX “cooperation”)
  - equivalently, ‘one cell’ with a distributed antenna array
- Macro-diversity can mitigate shadow fading[2] and increase capacity
- For N-transmitter, K-receiver system, i’s QoS given by:
  - $$\frac{P_i h_{i,1}}{Y_{i,1}+\sigma_1}+\cdots+\frac{P_i h_{i,K}}{Y_{i,K}+\sigma_K}$$
  - with $Y_{i,k}=\sum_{n\neq i} P_n h_{n,k}$
    P_n: power from transmitter n
    $h_{n,k}$: channel gain from transmitter n to receiver k

Two fundamental questions

- Each terminal “aims” for certain level of QoS, α_i
- With many terminals present, interference to a terminal grows with the power emitted by the others.
- Even without power limits, it is unclear that each terminal can achieve its desired QoS.
- Two fundamental questions:
  - Are the QoS targets feasible (achievable)? $\Leftarrow$CRITICAL for admission control!
  - If yes, which power vector achieves the QoS targets?

Main result

Fact

The vector α of QoS targets is feasible, if for each transmitter i at each receiver k,

$$\sum_{\substack{n=1\\n\neq i}}^{N} \alpha_n g_{n,k} < 1$$

where $g_{n,k} = h_{n,k} / \sum_k h_{n,k}$. The power vector that produces α can be found by successive approximations, starting from arbitrary power levels.

- Interpretation
  - Greatest *weighted* sum of $N-1$ QoS targets must be < 1
  - The weights are *relative* channel gains.
  - At most NK such simple sums need to be checked

Methodology: Fixed-point theory

- Power adjustment process $\Rightarrow$a *transformation* $\mathbf{T}$ that takes a power vector $\mathbf{p}$ and "converts" it into a new one, $\mathbf{T}(\mathbf{p})$.
- A limit of the process is a vector s.t. $\mathbf{p}^* = \mathbf{T}(\mathbf{p}^*)$; that is, a "fixed-point" of $\mathbf{T}$

Fact

(Banach's)If $\mathbf{T} : S \rightarrow S$ *is a contraction in* $S \subset \Re^M$ *(that is,* $\exists r \in [0, 1)$ *such that* $\forall \mathbf{x}, \mathbf{y} \in S, \|T(\mathbf{x}) - T(\mathbf{y})\| \leq r \|\mathbf{x} - \mathbf{y}\|$ *) then* $\mathbf{T}$ *has a unique fixed-point, that can be found by successive approximation, irrespective of the starting point [3]*

- We identify conditions under which the power-adjustment transformation is a contraction.

Methodology: key steps

- We replace
  - each $Y_{i,k}(\mathbf{P})$ with $\hat{Y}_i := \max_k\{Y_{i,k}\}$ and
  - each σ_k with $\hat{\sigma} := \max_k\{\sigma_k\}$.
- Then, the power adjustment takes the simple form

$$(h_i/\alpha_i)P_i^{t+1} = \hat{Y}_i(\mathbf{P}^t) + \hat{\sigma}$$

- We prove that $\hat{Y}_i := \max_k\{Y_{i,k}\} \equiv \|\mathbf{Y}_i(\mathbf{P})\|$ defines a “norm” on **P**. This allows us to invoke the “reverse” triangle inequality, which eventually leads to the result.

Original feasibility condition

- (Hanly, 1996 [1]) provides the condition

$$\sum_{n=1}^{N} \alpha_n < K$$

- Formula derived under certain simplifying assumptions:
  - A TX contributes to own interference
  - all TX's can be "heard" by all RX's
  - non-overcrowding
- Under certain practical situations condition is counter-intuitive:
  - If there are 2 TX near *each* RX, it must be "better", than if all TX's congregate near same receiver
  - In latter case, system should behave like a one-RX system
  - But formula is insensitive to channel gains: cannot adapt!

Special symmetric scenario

- Our condition is most similar to original when $h_{i,k} \approx h_{i,m}$ for all i, k, m, in which case $g_{i,k} \approx 1/K$
- Example: TX along a road; the axis of the 2 symmetrically placed RX is perpendicular to road
- Under this symmetry (and with $\alpha_N \leq \alpha_n \forall n$ for convenience) our condition simplifies to

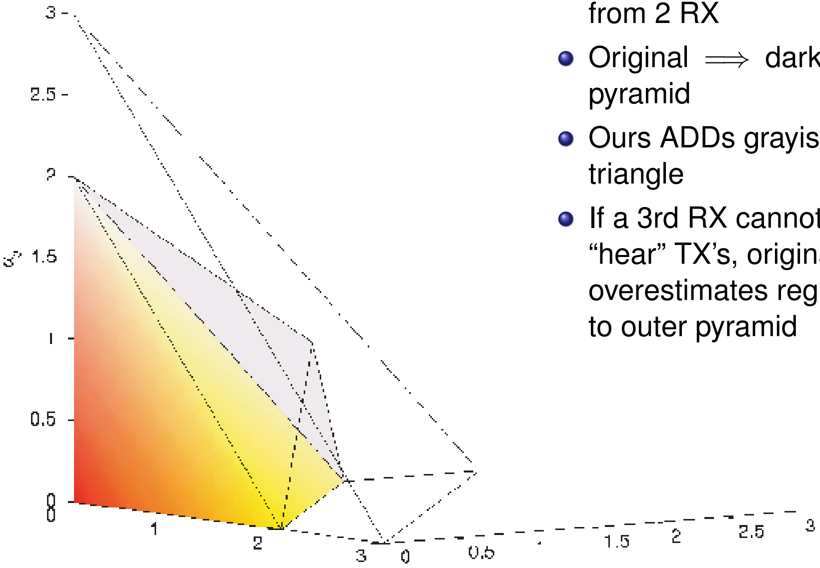
$$\sum_{n=1}^{N-1} \alpha_n < K$$

- Smallest α is left out of sum $\implies$ our condition is less conservative than original

Partial symmetry: one receiver “too far”

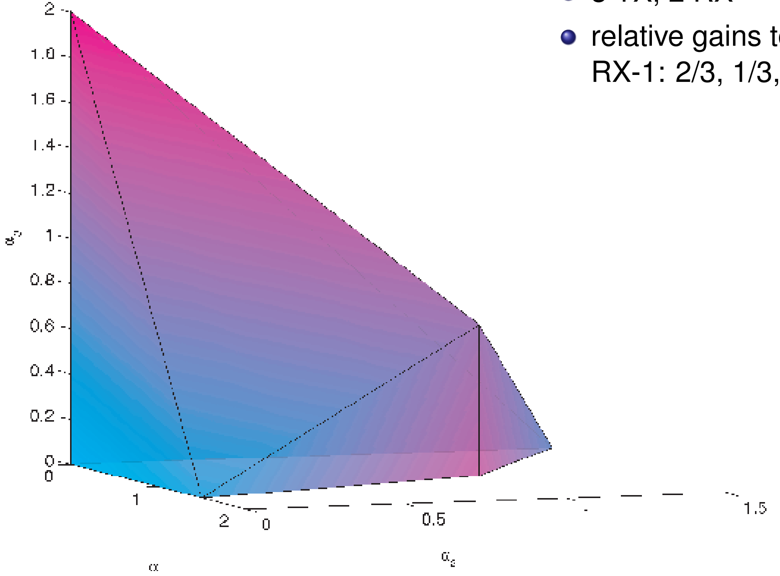
- If $K = 3$ and $h_{i,k} \approx h_{i,m}$ for all i, k, m, $g_{i,k} \approx 1/3$ and our condition becomes $\sum_{n=1}^{N-1} \alpha_n < 3$
- But suppose that $h_{i,1} \approx h_{i,2}$ but $h_{i,3} \approx 0$ (3rd receiver is “too far”), then $g_{i,3} \approx 0$ and $g_{i,1} \approx g_{i,2} \approx 1/2$
- Thus our condition leads to $\sum_{n=1}^{N-1} \alpha_n < 2$
- Our condition automatically “adapts”, whereas original remains at $\sum_{n=1}^{N} \alpha_n < 3$
- Original can over-estimate capacity if applied when some RX’s are “out of range” (because under this situation — of practical interest — some assumptions underlying the original are not satisfied)

Symmetric 3TX, 2RX scenario



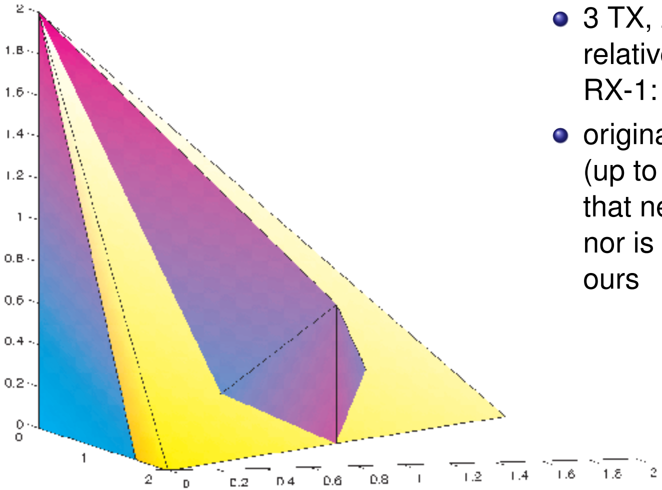
- 3 TX "equidistant" from 2 RX
- Original $\Longrightarrow$ darker pyramid
- Ours ADDs grayish triangle
- If a 3rd RX cannot "hear" TX's, original overestimates region to outer pyramid

Asymmetric 3TX, 2RX scenario: our region



- 3 TX, 2 RX
- relative gains to RX-1: 2/3, 1/3, 1/2

Asymmetric 3TX, 2RX: ours vs original



- 3 TX, 2 RX with relative gains to RX-1: 2/3, 1/3, 1/2
- original yields region (up to yellow volume) that neither contains nor is contained by ours

Recapitulation

- With macro-diversity receivers “cooperate” in decoding each TX
- Scheme can mitigate shadow fading and increase capacity
- Original feasibility formula may overestimate capacity under certain practical situations (e.g. a given TX is in a range of only a few RX)
- On the foundation of Banach’ fixed-point theory, a new formula has been derived that,
  - is only slightly more complex than original,
  - adjusts itself — through a dependence on relative channel gains – to non-uniform geographical distributions of TX
  - leads to a practical admission-control algorithm (see paper)
- Analysis has been extended to other practical schemes, and to a generalised multi-receiver radio network

Generalised multi-receiver radio network

- Analysis extended to a generalised radio network
- *i*'s QoS requirement given by

$$\mathscr{Q}_i\left(\frac{P_i h_{i,1}}{\mathscr{Y}_{i,1}(\mathbf{P})+\sigma_1},\cdots,\frac{P_i h_{i,K}}{\mathscr{Y}_{i,K}(\mathbf{P})+\sigma_K}\right)\geq\alpha_i$$

- $\mathscr{Q}_i$, and $\mathscr{Y}_{i,k}$ are general functions obeying certain simple properties (monotonicity, homogeneity, etc)
- For macro-diversity
  - $\mathscr{Y}_{i,k}(\mathbf{P})=\sum_{n\neq i}P_n h_{n,k}$
  - $\mathscr{Q}_i(\mathbf{x})=\mathscr{Q}(\mathbf{x})=x_1+\cdots+x_K$ (same function works for all *i*)
- Feasibility results obtained for multiple-connection reception and all other scenarios of (Yates, 1995) ([4])
- See IEEE-WCNC, 5-8 April 2009, Budapest

Questions?

Norms I

Let V be a vector space (see [5, pp. 11-12] for definition).

Definition

A function $f\colon V \to \Re$ is called a *semi-norm* on V, if it satisfies:

1. $f(v) \geq 0$ for all $v \in V$ (non-negativity)
2. $f(\lambda v) = |\lambda| \cdot f(v)$ for all $v \in V$ and all $\lambda \in \Re$ (homogeneity)
3. $f(v + w) \leq f(v) + f(w)$ for all $v, w \in V$ (*triangle ineq.*)

Definition

If f also satisfies $f(v) = 0 \iff v = \theta$ (where θ is the zero element of V), then f is called a *norm* and $f(v)$ is denoted as $||v||$

Norms II

Definition

The Hölder norm with parameter $p \geq 1$ ("p-norm") is denoted as $||\cdot||_p$ and defined for $x \in \Re^N$ as $\|\mathbf{x}\|_p = (|x_1|^p + \cdots + |x_N|^p)^{\frac{1}{p}}$

With $p = 2$, the Hölder norm becomes the familiar Euclidean norm. Also, $\lim_{p\to\infty} \|\mathbf{x}\|_p = \max(|x_1|, \cdots, |x_N|)$, thus:

Definition

For $x \in \Re^N$, the infinity or "max" norm is defined by $\|\mathbf{x}\|_\infty := \max(|x_1|, \cdots, |x_N|)$

Banach fixed-point theorem

Definition

A map T from a normed space $(V, \|\cdot\|)$ into itself is a *contraction* if there exists $r \in [0,1)$ such that for all $x, y \in V$, $\|T(x) - T(y)\| \leq r\|x - y\|$

Theorem

(Banach' Contraction Mapping Principle) If T is a contraction mapping on V there is a unique $x^* \in V$ such that $x^* = T(x^*)$. Moreover, x^* can be obtained by successive approximation, starting from an arbitrary initial $x_0 \in V$. [3]

For Further Reading I

- S. V. Hanly, “Capacity and power control in spread spectrum macrodiversity radio networks,” *Communications, IEEE Transactions on*, vol. 44, no. 2, pp. 247–256, Feb 1996.
- E. B. Bdira and P. Mermelstein, “Exploiting macrodiversity with distributed antennas in micro-cellular CDMA systems,” *Wireless Personal Communications*, vol. 9, no. 2, pp. 179–196, 1999.

For Further Reading II

- S. Banach, *Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales*.
  PhD thesis, University of Lwów, Poland (now Ukraine), 1920.
  Published: Fundamenta Mathematicae 3, 1922, pages 133-181.
- R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on Selected Areas in Communications*, vol. 13, pp. 1341–1347, Sept. 1995.
- D. Luenberger, *Optimization by vector space methods*.
  New York: Wiley, 1969.