A generalised multi-receiver radio network and its decomposition into independent transmitter-receiver pairs: Simple feasibility condition and power levels in closed form

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Outline

Power, interference and QoS: 2 questions

- In many interesting situations, user's QoS increases with the power in its signal, and decreases with the interfering power present at the concerned receiver(S)
- Typically each terminal "aims" for certain level of QoS
- Two fundamental questions:
  - Are the QoS targets feasible (achievable)? $\Leftarrow$CRITICAL for admission control!
  - If yes, which power vector achieves the QoS targets?
- Ideally, one would like to answer these questions for a generallised network that includes many past, present, and future networks as special cases.

Abstract model (Yates'95)

- N terminals whose power choices affect each other
- Terminal i chooses a power p_i given by a function $g_i(\mathbf{p}_{-i})$, with $\mathbf{p}_{-i}$ denoting the power levels of the others
- $p_i = g_i(\mathbf{p}_{-i})$ leads to terminal i its desired QoS for given $\mathbf{p}_{-i}$
- All details of the network (the QoS targets, number of receivers, interference functions, etc) are assumed "hidden" inside the power functions
- These functions are assumed to satisfy some simple mathematical properties (monotonicity, homogeneity, etc)
- Considering the functions properties the analyst addresses some of the fundamental questions about QoS achievability[1]

Generalised multi-receiver radio network

- N transmitters, K receivers
- i's QoS requirement given by

$$\mathcal{Q}_i\left(\frac{P_i h_{i,1}}{\mathcal{Y}_{i,1}(\mathbf{P})+\sigma_1},\cdots,\frac{P_i h_{i,K}}{\mathcal{Y}_{i,K}(\mathbf{P})+\sigma_K}\right)\geq\kappa_i \qquad (1)$$

- $h_{i,k}$ is the known channel gain from TX i to RX k
- $\mathcal{Q}_i$, and $\mathcal{Y}_{i,k}$ are general functions obeying certain simple properties (monotonicity, homogeneity, etc)

An example: macro-diversity

- macro-diversity:
  - definition
    - cellular structure is removed
    - all transmitters are jointly decoded by all receivers
    - equivalently, 'one cell' with a distributed antenna array
  - i's QoS is given by [2]:
    - $P_i h_{i,1}/(Y_{i,1}+\sigma_1)+\cdots+P_i h_{i,K}/(Y_{i,K}+\sigma_K)$
    - with $Y_{i,k}=\sum_{n\neq i} P_n h_{n,k}$
  - Thus, $\mathscr{Y}_{i,k}(\mathbf{P})=\sum_{n\neq i} P_n h_{n,k}$ and
  - $\mathscr{Q}_i(\mathbf{x})=\mathscr{Q}^{\mathrm{MD}}(\mathbf{x})=x_1+\cdots+x_K$
    (notice that same function works for all i)
- Other examples: all scenarios from (Yates 1995)[1]

Motivation: Why a new model?

- Both models can be useful (think macroeconomics vs. microeconomics)
- Abstract model is more general (powerful?)
- Detailed model
  - is closer to 'real' world (easier to interpret)
  - separates QoS function from Interference function (conceptually different... may have different properties)
  - may provide insights/opportunities not otherwise available (e.g., we provide a simple closed-form solution for this model... see below)

Main result

Let κ_i denote i's QoS target, and
$q_i = \mathscr{Q}_i \left(h_{i,1}/\mathscr{Y}_{i,1}(\mathbf{1}), \cdots, h_{i,K}/\mathscr{Y}_{i,K}(\mathbf{1}) \right) \Leftarrow$ QoS with each power level equal to unity.

Theorem

If the functions $\mathscr{Q}_i$ and $\mathscr{Y}_{i,k}$ are non-negative, non-decreasing, and homogeneous , and additionally, random noise is negligible, then $\kappa_i \leq q_i \, \forall i$ implies that (i) each QoS target can be achieved, in particular, (ii) with the power levels $P_i^* = \kappa_i / q_i$.

Network simplification

Consider 'network' with N independent (orthogonal) transmitter-receiver pairs.
Each transmitter has a power limit $\bar{P}_i = \sigma_i := 1$ and wants QoS (SNR) of κ_i.
Let the channel gain of transmitter i be $h_i := q_i$.

- The maximal QoS that i can achieve is $\bar{P}_i h_i / \sigma_i = h_i = q_i$.
- Thus κ_i is achievable provided $\kappa_i \leq q_i$.
- Furthermore, if $\kappa_i / q_i \leq 1$ then $P_i = \kappa_i / q_i$ is feasible ($\leq \bar{P}_i = 1$), and yields an SNR exactly equal to κ_i.
- The "solution" to this simple 'network´ works for the original one!

A simple and useful Lemma

Let $f : \Re^M \rightarrow \Re$, and $\mathbf{1}_M$ denote the "all ones" M-vector.

Definition

f if *positively quasi-homogeneous* (of degree one) if for all $r \in \Re_+$, $f(r\mathbf{1}) = rf(\mathbf{1})$

Definition

f if *quasi-non-decreasing if* $f(\mathbf{x}) \leq f(\|\mathbf{x}\|\mathbf{1})$, where $\|\mathbf{x}\|$ denotes the largest absolute value of the components of $\mathbf{x}$.

Fact

If f satisfies both definitions, $f(\mathbf{x}) \leq f(\|\mathbf{x}\|\mathbf{1}) = \|\mathbf{x}\|f(\mathbf{1})$

Solution applied to macro-diversity

- For macro-diversity, $\mathscr{Y}_{i,k}(\mathbf{1}) = \sum_{n \neq i} h_{n,k}$.
- Since $\mathscr{Q}_i^{\mathrm{MD}}(\mathbf{x}) = x_1 + \cdots + x_K$, then

$$q_i^{\mathrm{MD}} = \sum_{k=1}^{K} \frac{h_{i,k}}{\sum_{n \neq i} h_{n,k}}$$

- Thus, the feasibility condition is $\kappa_i \leq q_i^{\mathrm{MD}}$ and a solution is $P_i = \kappa_i / q_i^{\mathrm{MD}}$
- If all $h_{i,k}$ are of the same order of magnitude $q_i^{\mathrm{MD}} \approx \sum_{k=1}^{K} 1/(N-1) = K/(N-1)$
- Then the condition becomes $\kappa_i \leq K/(N-1)$
- Thus, $\sum_{k=1}^{N} \kappa_i \leq KN/(N-1) \approx K$

Other macro-diversity formulae

- (Hanly, 1996 [2]) provides the condition

$$\sum_{n=1}^{N} \kappa_n < K$$

- (Rodriguez, et al., 2008 [3]) provides a condition that — with each transmitter "equidistant to each receiver (and with $\kappa_N \leq \kappa_n \forall n$ for convenience) — simplifies to:

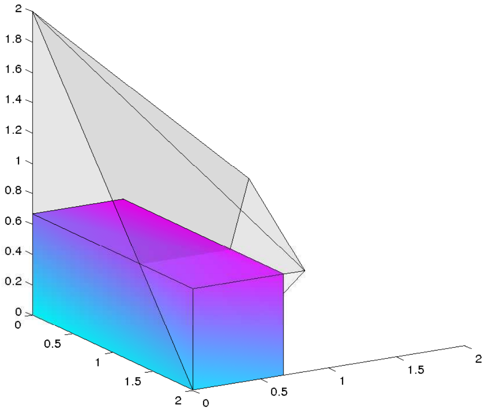
$$\sum_{n=1}^{N-1} \kappa_n < K$$

Macro-diversity formulae compared

Table: Macro-diversity formulae under symmetry

Herein	Rodriguez08	Hanly96
$\sum_{k=1}^{N} \kappa_i \leq KN/(N-1)$	$\sum_{n=1}^{N-1} \kappa_n < K$	$\sum_{n=1}^{N} \kappa_n < K$

Macro-diversity achievable regions



- TX 1, 2 & 3 at (0,0), (-1,0), (1,0)
- RX 1, 2 are at (0,-1), and (0,1)
- $h_{i,k} \propto d_{i,k}^{-2}$ with $d_{i,k}$ the distance from i to k
- $d_{1,k} = 1$; $d_{2,k} = d_{3,k} = \sqrt{2}$
- $h_{1,1} = h_{1,2} \propto 1$; $h_{2,k} = h_{3,k} \propto 1/2$

Recapitulation: strengths

- Model seems to be new
- Explicit (conservative) feasibility condition given ($\kappa_i \leq q_i$)
- Matching power vector given ($P_i = \kappa_i / q_i$)
- Interpretation: Generalised radio network can be (conservatively) associated with set of independent transmitter receiver pairs
- Analysis already extended to consider noise (Submitted)
- Solution is technology/application independent (useful for present and future networks)
- Analysis BOTH generalises AND simplifies (these are usually contrary aims)
- Provides specific/detailed information (formulae) applicable to wide variety of networks (result-specificity and result-generality tend to be contrary aims)

Recapitulation: limitations and outlook

- To consider SIC, $\mathscr{Q}_i$ must be made non-monotonic in interference. Rest of the model is OK.
- How “conservative” is the solution?
- Channel gains are assumed deterministic: can/should they be considered as random variables?
- Homogeneity plays a key role: Can it be removed, so that only monotonicity remains?
- Can/should media-based communication (e.g. video) be explicitly considered (e.g. through $\mathscr{Q}_i$)?

Questions?

Argument I

Let $\mathbf{P}$ denote power (and suppose $K = 2$). $(P_i / \|\mathbf{P}\|) q_i \geq \kappa_i \equiv$

- $$(P_i / \|\mathbf{P}\|) \mathscr{Q}_i \left(\frac{h_{i,1}}{\mathscr{Y}_{i,1}(\mathbf{1})}, \frac{h_{i,2}}{\mathscr{Y}_{i,2}(\mathbf{1})} \right) \geq \kappa_i \quad \Longrightarrow$$
  $$\mathscr{Q}_i \left(\frac{P_i h_{i,1}}{\|\mathbf{P}\| \mathscr{Y}_{i,1}(\mathbf{1})}, \frac{P_i h_{i,2}}{\|\mathbf{P}\| \mathscr{Y}_{i,2}(\mathbf{1})} \right) \geq \kappa_i \qquad \text{(by homogeneity)}$$
- $\mathscr{Y}_{i,k}(\mathbf{P}) \leq \|\mathbf{P}\| \mathscr{Y}_{i,k}(\mathbf{1})$ (key Fact), thus
- $$\mathscr{Q}_i \left(\frac{P_i h_{i,1}}{\mathscr{Y}_{i,1}(\mathbf{P})}, \frac{P_i h_{i,2}}{\mathscr{Y}_{i,2}(\mathbf{P})} \right) \geq \mathscr{Q}_i \left(\frac{P_i h_{i,1}}{\|\mathbf{P}\| \mathscr{Y}_{i,1}(\mathbf{1})}, \frac{P_i h_{i,2}}{\|\mathbf{P}\| \mathscr{Y}_{i,2}(\mathbf{1})} \right)$$
- $\therefore$ if $(P_i / \|\mathbf{P}\|) q_i \geq \kappa_i$ or $P_i / \|\mathbf{P}\| \geq \kappa_i / q_i$, each κ_i is reached or exceeded

Argument II

- But $P_i \leq \|\mathbf{P}\| \ \forall i$, for any $\mathbf{P}$, by definition.
- Therefore, *no* power vector can satisfy $P_j / \|\mathbf{P}\| \geq \kappa_j / q_j > 1$
- With $\hat{\kappa} := (\kappa_1/q_1, \cdots, \kappa_N/q_N) := (\hat{\kappa}_1, \cdots, \hat{\kappa}_K)$,
  $\hat{\kappa}_i = \kappa_i/q_i \leq 1 \ \forall i \implies \|\hat{\kappa}\| \leq 1 \implies \hat{\kappa}_i / \|\hat{\kappa}\| \geq \hat{\kappa}_i \ \forall i$
- $\therefore \mathbf{P}^* = \hat{\kappa}$ satisfies $P_i / \|\mathbf{P}\| \geq \kappa_i / q_i \ \forall i$ and yields or exceeds the desired QoS

For Further Reading

- R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on Selected Areas in Communications*, vol. 13, pp. 1341–1347, Sept. 1995.
- S. V. Hanly, "Capacity and power control in spread spectrum macrodiversity radio networks," *Communications, IEEE Transactions on*, vol. 44, no. 2, pp. 247–256, Feb 1996.
- V. Rodriguez, R. Mathar, and A. Schmeink, "Capacity and power control in spread spectrum macro-diversity radio networks revisited," in *IEEE Australasian Telecommunications Networking and Application Conference*, 2008.